for any $x > 0$ and all sufficiently large $n$, where $A$ is an absolute positive constant. By condition (6) we have $a_n = n$ for all sufficiently large $n$ if the constant $\varepsilon$ is chosen sufficiently small. For these values of $n$ we have $\psi(n) = \psi(n)$, since the function $\psi$ is nondecreasing. Hence, condition (1) is satisfied with the sum $S_n = \sum_{j=1}^{n} X_j$ replaced by $|S_n|$. Therefore, by Theorem 1 we obtain

$$\lim \sup \frac{|S_n|}{a_n} s.t a_n$$

(11)

In the definition of $a_n$ we may choose the number $\varepsilon$ arbitrarily small, and therefore (9) follows from (11).

We note a corollary of Theorem 2. Suppose that $\{X_n\}$ is a random sequence which is stationary in the broad sense with $E X_n = 0$. If the series $\sum_{j=0}^{\infty} |r_j|$ converges, then

$$S_n = o\left(\frac{1}{\varepsilon \log n}\right) n.s.$$

for any function $\psi \in \Psi_C$.

Theorem 4 of the work [3] follows from the last assertion.

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INFORMATION IN A SCHEME WITH ADDITIVE NOISE

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A formula is proved expressing the information contained in a stationary, linearly regular, Gaussian process with an independent additive increment relative to the original unperturbed process.

1. Let $y$ and $z$ be stationary, linearly regular, Gaussian processes with discrete or continuous time and spectral densities (s.d.) $f_y$ and $f_z$, respectively; let

$$\bar{x}(t) = y(t) + z(t).$$

(1)

We shall consider the scheme with the additive "noise" $z$, i.e., we assume that the processes $y$ and $z$ are independent. In order to eliminate trivial complications, we assume that $Ey(t) = Ez(t) = 0$. If a process $u(t), t \in \Omega \subset \mathbb{R}$, is given, we denote by $H_u$ the complex Hilbert space generated by the quantities $u(t), t \in \Omega$, with scalar product $(\xi, \eta) = \int_{\Omega} \eta(t) \overline{\xi(t)} dt$ and by $U_u$ the subspace of the Hilbert space $H_u$ constructed on the basis of the quantities $u(t), t \in [a, b] \cap \Omega$.

We consider the amount of information $J_t = J_t(x, y) \{1\}$ contained in the quantities $x_t$ relative to the quantities $y_t$. It is known that for processes with discrete time

$$J(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left| \frac{f_x}{f_z} \right| \frac{f_x}{f_z} \left| f_{xy} \right| d\lambda,$$

(2)

where \( f_{xy} \) is the mutual spectral density of the pair \((x, y)\),

\[
R_{xy}(\tau) \overset{df}{=} \mathbb{E} x(t + \tau) y(t) = \int_{\mathbb{R}} e^{i\tau \lambda} f_{xy} \, d\lambda.
\]

In our case, that is assuming the independence of the processes \( y \) and \( z \), \( f_{xy} = f_y \). An analogue of formula (2) in the case where the spectrum of the pair \((x, y)\) is rational has been established in [1]. The general case is considered in [2]. Namely, it was established that if at least one of the processes \( x \) or \( y \) is weakly regular (the process \( x \) is weakly regular if \( \|C_{2I}L \| < \infty \) for any \( t < \infty \)), then for processes with continuous time

\[
\psi(x, y) = \frac{1}{2\pi \int_{\mathbb{R}} \ln \frac{|f_{xy}|}{|f_x f_y|} e^{-t} \, d\lambda}.
\]

In the present note conditions are found under which there exists the finite limit \( \lim_{t \to \infty} \psi(x, y) \) and the value of this limit is computed.

2. Let \( x(n) \) be a stationary, linearly regular, Gaussian sequence with s.d. \( f \). We compute the quantity

\[
\chi \overset{df}{=} \chi(x^{\rightarrow}, x^{\leftarrow}), \text{ i.e., the amount of information contained in the "past" of the process } x \text{ with respect to its "future."}
\]

Let \( g \) be an outer function of the space \( H^2 \) of the interior of the disk [3] such that \( |g(z)|^2 = f(z) \) almost everywhere on the unit circle. The possibility of such a factorization of the s.d. \( f \) follows from the condition of linear regularity of the process \( x \) [4]. The following theorem holds.

**Theorem 1.** There is the formula

\[
\chi = \frac{1}{2\pi \int_{\mathbb{R}} \ln \frac{|Q(z)|^2}{|Q(\bar{z})|^2} \, d\lambda} = \frac{1}{2} \sum_{k} |h_k|^2 \chi,
\]

where \( d\lambda \) is Lebesgue measure in the plane, \( h_k \) are the Fourier coefficients of the function \( \ln f \), \( \ln f(z) = \sum h_k z^k \) (if the process \( x \) is linearly regular).

**Proof.** We denote by \( P^- \), \( P^+ \) the orthoprojectors in \( X_i \) onto the subspaces \( X_i \), \( X_i \), respectively. Then [1] since the process \( x \) is Gaussian,

\[
\chi = \chi(x^{\rightarrow}, x^{\leftarrow}) = \frac{1}{2} \ln \det (E - P^- P^+ P^-),
\]

where \( E \) is the identity operator. Let \( Vx(n) = x(n) \). It is known [4] that the mapping extends to an isometry of \( \chi \) onto the \( L^2 \) space on the unit circle constructed on the basis of the measure \( f d\lambda \). The latter space we denote by \( L^2_\chi \). Let \( H_f^+ \), \( H_f^- \) be the subspaces of the space \( L^2_\chi \) generated by the functions \( \{Z^n, n < 0\} \), \( \{Z^n, n > 0\} \), respectively, and let \( P_f^+ \), \( P_f^- \) be the orthoprojectors in \( L^2 \) onto the subspaces \( H_f^+ \), \( H_f^- \). Since \( Vh^+ = H_f^+ \), \( Vh^- = H_f^- \), by the isometric property of the mapping

\[
d_f = \det (E - P_f^- P_f^+ P_f^-) = \det (E - P_f^- P_f^+ P_f^-),
\]

It was established in [5] that

\[
d_f = \mathbb{E} \exp \left\{ -\frac{1}{2} \int_{\mathbb{R}} \ln \frac{|Q(z)|^2}{|Q(\bar{z})|^2} \, d\lambda \right\} = \mathbb{E} \exp \left\{ -\sum_{k=1}^\infty |h_k|^2 \chi \right\}.
\]

This and formulas (5) and (4) imply (3). The proof of the theorem is complete.

Let \( x \) be a generalized Gaussian stationary process with continuous time [6], i.e., \( x \) is a continuous random functional on the space \( \Phi \) of infinitely differentiable functions such that

1) the random variables \( x(\varphi), \varphi \in \Phi \), are Gaussian;

2) \( m_x(\varphi) \overset{df}{=} \mathbb{E} x(\varphi) = m_x(\tau \varphi, \varphi) \);

3) \( R_x(\varphi, \psi) \overset{df}{=} \mathbb{E} x(\varphi) \overline{x(\psi)} = R_x(\tau \varphi, \tau \psi) \), where \( \tau \) is the shift operator: \( |\tau \varphi| = \varphi(\xi + \psi) \).

We assume that the process \( x \) is defined only on the subspace \( \Phi_X \) of the space \( \Phi \) distinguished by the condition of finiteness of the quantities \( E|x(\varphi)|^2 \), \( \varphi \in \Phi_X \). Let \( \chi \) be the complex Hilbert space generated by the quantities \( x(\varphi), \varphi \in \Phi_X \), with scalar product \( \langle \xi, \psi \rangle = E \xi_\varphi \overline{\psi} \), and let \( \chi_{\Phi_X} \) be the subspace of the space \( \chi \).