for any $x > 0$ and all sufficiently large $n$, where $A$ is an absolute positive constant. By condition (8) we have $a_n = n$ for all sufficiently large $n$ if the constant $\epsilon$ is chosen sufficiently small. For these values of $n$ we have $\psi(a_n) \leq \psi(n)$, since the function $\psi$ is nondecreasing. Hence, condition (1) is satisfied with the sum $S_k = \sum_{j=1}^{k} X_j$ replaced by $|S_k|$: therefore, by Theorem 1 we obtain

$$\lim_{n \to \infty} \sup \frac{|S_n|}{a_n} = 1. \quad (11)$$

In the definition of $a_n$ we may choose the number $\epsilon$ arbitrarily small, and therefore (9) follows from (11).

We note a corollary of Theorem 2. Suppose that $\{X_n\}$ is a random sequence which is stationary in the broad sense with $EX_n = 0$. If the series $\sum_{j=1}^{\infty} \mu_j$ converges, then

$$S_n = 0 \left( \frac{n \psi(n)}{\log n} \right) \text{ a.s.}$$

for any function $\psi \in \Psi_c$.

Theorem 4 of the work [3] follows from the last assertion.

**LITERATURE CITED**

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**INFORMATION IN A SCHEME WITH ADDITIVE NOISE**

V. N. Solev

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A formula is proved expressing the information contained in a stationary, linearly regular, Gaussian process with an independent additive increment relative to the original unperturbed process.

1. Let $y$ and $z$ be stationary, linearly regular, Gaussian processes with discrete or continuous time and spectral densities (s.d.) $f_y$ and $f_z$, respectively; let

$$x(t) = y(t) + z(t). \quad (1)$$

We shall consider the scheme with the additive "noise" $z$, i.e., we assume that the processes $y$ and $z$ are independent. In order to eliminate trivial complications, we assume that $Ey(t) = Ez(t) = 0$. If a process $u(t)$, $t \in [\alpha, \beta] \cap I$, is given, we denote by $\mathcal{H}_u$ the complex Hilbert space generated by the quantities $u(t)$, $t \in [\alpha, \beta]$, with scalar product $(\xi, \eta) = \int_{[\alpha, \beta]} \xi(t)\eta(t) dt$ and by $\mathcal{U}_u$ the subspace of the Hilbert space $\mathcal{H}_u$ constructed on the basis of the quantities $u(t)$, $t \in [\alpha, \beta] \cap I$.

We consider the amount of information $\mathcal{J}_t = \mathcal{J}_t(x, y) \{1\}$ contained in the quantities $x_t$ relative to the quantities $y_t$. It is known that for processes with discrete time

$$\mathcal{I}(x, y) \stackrel{d}{=} \lim_{t \to \infty} \frac{\mathcal{J}_t}{t} = \frac{1}{2\pi} \int_{[\alpha, \beta]} \ln \frac{f_x f_y}{f_x f_y} \left| \int f_x f_y \right| \, dx, \quad (2)$$

where $f_{xy}$ is the mutual spectral density of the pair $(x, y)$,

$$R_{xy}(\tau) \triangleq \mathbb{E} x(t + \tau) \mathbb{E}^* y(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{i\tau \xi} f_{xy} \, d\xi.$$ 

In our case, that is assuming the independence of the processes $y$ and $z$, $f_{xy} = f_y$. An analogue of formula (2) in the case where the spectrum of the pair $(x, y)$ is rational has been established in [1]. The general case is considered in [2]. Namely, it was established that if at least one of the processes $x$ or $y$ is weakly regular (the process $x$ is weakly regular if $\sum_{n=1}^{\infty} n \mathbb{E} |x_n|^2 < \infty$ for any $t < \infty$), then for processes with continuous time

$$i(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \ln \left( \frac{f_x f_y}{f_{xy}} \right) \, d\xi.$$  

In the present note conditions are found under which there exists the finite limit $\lim_{t \to \infty} \left[ J_t - t i(x, y) \right]$ and the value of this limit is computed.

2. Let $x(n)$ be a stationary, linearly regular, Gaussian sequence with s.d. $f$. We compute the quantity $J_x \triangleq \mathbb{E}(x^{-1}, x^n)$, i.e., the amount of information contained in the "past" of the process $x$ with respect to its "future." Let $g$ be an outer function of the space $H^2$ of the interior of the disk [3] such that $|g(z)|^2 = f(z)$ almost everywhere on the unit circle. The possibility of such a factorization of the s.d. $f$ follows from the condition of linear regularity of the process $x$ [4]. The following theorem holds.

**THEOREM 1.** There is the formula

$$J_x = \frac{1}{2\pi} \int_{|\xi|<1} \left| \frac{Q(\xi)}{q(\xi)} \right|^2 \, d\xi = \frac{1}{2\pi} \sum_{k \in \mathbb{Z}} |h_k|^2 \chi,$$  

where $d\xi$ is Lebesgue measure in the plane, $h_k$ are the Fourier coefficients of the function $\ln f$, $\ln f(\xi) = \sum_{k \in \mathbb{Z}} h_k \chi^n$ ($\ln f \in L^1$ if the process $x$ is linearly regular).

**Proof.** We denote by $P^-, P^+$ the orthoprojectors in $\mathbb{X}^{-\infty}$ onto the subspaces $\mathbb{X}^{-\infty}$, $\mathbb{X}^\infty$, respectively. Then since the process $x$ is Gaussian,

$$J_x = J(\mathbb{X}^{-\infty}, \mathbb{X}^\infty) = -\frac{1}{2\pi} \ln \det (E - P^- P^+ P^-),$$  

where $E$ is the identity operator. Let $V|\mathbb{X}(n)| = \mathbb{Z}^n$. It is known [4] that the mapping extends to an isometry of $\mathbb{X}^{-\infty}$ onto the $L^2$ space on the unit circle constructed on the basis of the measure $f \, d\lambda$. The latter space we denote by $L^2_f$. Let $H_f^+, H_f^-$ be the subspaces of the space $L^2_f$ generated by the functions $\{Z_n, n < 0\}$, $\{Z_n, n \geq 0\}$, respectively, and let $P_f^-, P_f^+$ be the orthoprojectors in $L^2_f$ onto the subspaces $H_f^+, H_f^-$. Since $VH^- = H_f^-, VH^+ = H_f^+$, by the isometric property of the mapping $V$

$$d_f = \det (E - P_f^- P_f^+ P_f^-) = \det (E - P_j P^+ P_j).$$  

It was established in [5] that

$$d_f = \exp \left\{ -\frac{1}{4\pi} \int_{|\xi|<1} \left| \frac{Q(\xi)}{q(\xi)} \right|^2 \, d\xi \right\} \exp \left\{ -\sum_{k \in \mathbb{Z}} |h_k|^2 \chi \right\}.$$  

This and formulas (5) and (4) imply (3). The proof of the theorem is complete.

Let $x$ be a generalized Gaussian stationary process with continuous time [6], i.e., $x$ is a continuous random functional on the space $\Phi$ of infinitely differentiable functions such that

1) the random variables $x(\varphi)$, $\varphi \in \Phi$, are Gaussian;

2) $m_x(\varphi) \triangleq \mathbb{E} x(\varphi) = m_x(\tau_t \varphi)$;

3) $R_x(\varphi, \varphi) \triangleq \mathbb{E} x(\varphi) x(\varphi) = R_x(\tau_t \varphi, \tau_t \varphi)$, where $\tau_t$ is the shift operator: $[\tau_t \varphi](s) = \varphi(t + s)$.

We assume that the process $x$ is defined only on the subspace $\Phi_x$ of the space $\Phi$ distinguished by the condition of finiteness of the quantities $E |x(\varphi)|^2$, $\varphi \in \Phi_x$. Let $\mathbb{X}$ be the complex Hilbert space generated by the quantities $x(\varphi)$, $\varphi \in \Phi_x$, with scalar product $(\xi, \xi') = E \xi \bar{\xi}'$, and let $\mathbb{X}_\alpha$ be the subspace of the space $\mathbb{X}$.