A Linearized Theory of Tube Drawing\(^1\)

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1. Introduction

The oldest yield condition in the theory of plasticity is that of TRESCA \(^1\).\(^2\). When the principal stresses are used as rectangular co-ordinates in a three-dimensional stress space, it is represented by a regular hexagonal prism. As long as the variations of stress in a problem are such that the stress point remains restricted to one face of this prism, the linear character of the yield condition greatly simplifies the mathematical analysis. The complications that arise when several faces must be considered in the solution of a problem induced VON MISES \(^2\) to suggest that the prism be approximated by the incised circular cylinder. Later experiments showed that the resulting non-linear yield condition, which was introduced for mathematical convenience, actually describes the behaviour of plastic metals better than the piecewise linear condition of TRESCA. On the other hand, the hope that a single non-linear expression would be less unwieldy than six linear expressions did not, on the whole, prove justified, and linearised yield conditions have been used in the solution of many problems in plasticity.

In the early stages of plasticity, yield condition and flow rule were viewed as independent components of the theory, and investigators felt free to use the flow rule of SAINT VENANT and Lévy \(^3\) with whatever yield condition proved convenient in the solution of a problem. Recent work on limit analysis, however, has suggested the connection between yield condition and flow rule that is expressed by the principle of maximum plastic dissipation \(^4\). If this principle is accepted, linearisation of the yield condition should be accompanied by a change in flow rule that maintains the validity of this principle. While this ‘joint linearisation of yield condition and flow rule’ has been widely used in limit analysis and design \(^5\), relatively few problems of finite plastic flow have been treated in this manner. In this paper, a problem of this kind will be solved for a material obeying the yield condition of VON MISES and the flow rule of SAINT VENANT and Lévy, and the solution will be compared to those obtained by various joint linearisations of yield condition and flow rule.

2. The Problem

In considering the drawing of a rigid, perfectly plastic thin-walled tube through a well-lubricated conical die (Figure 1), we shall restrict the discussion to steady states

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\(^1\) The results communicated in this paper were obtained in the course of research sponsored by the Office of Naval Research under Contract No. 562(10) with Brown University.

\(^2\) Numbers in brackets refer to References, page 12
of plastic flow and assume that the wall thickness is small in comparison to the length of contact with the die. The variation of stress across the thickness of the wall will therefore be neglected. On account of the rotational symmetry of the problem and the absence of shearing stresses between tube and die, the normals to the meridional planes and to the die surface indicate principal directions of stress.

![Figure 1](image)

Figure 1

Drawing of a thin tube through a conical die.

- $h_0$: initial thickness,
- $h$: thickness of tube at radius $r$,
- $b$: radius of tube at die exit,
- $a$: radius of tube at die entrance, taken as unit of length,
- $\sigma_1$: meridional stress.

Subject to later confirmation by the numerical results, it will be assumed that the normal pressure $p$ between the tube and die is small in comparison to the meridional stress $\sigma_1$ or the circumferential stress $\sigma_2$, so that a yield condition for plane stress and the corresponding flow rule will be appropriate. Since several yield conditions will be used, we shall establish the basic equations for an arbitrary yield condition written as

$$F(\sigma_1, \sigma_2) = 0. \quad (2.1)$$

In view of the rotational symmetry of the problem, and since the variations of $\sigma_1$ and $\sigma_2$ across the wall thickness are neglected, these principal stresses depend only on the distance $s$ measured along the die. With the notations in Figure 1, equilibrium in the meridional direction requires that

$$\frac{d}{ds} (\sigma_1 h r) + \sigma_2 h \sin \alpha = 0. \quad (2.2)$$

Since

$$s = \frac{a - r}{\sin \alpha}, \quad (2.3)$$

we may alternatively consider $\sigma_1$, $\sigma_2$ and $h$ as functions of $r$ to write (2.2) in the form

$$\frac{r}{h} \frac{dh}{dr} \sigma_1 + r \frac{d\sigma_1}{dr} + (\sigma_1 - \sigma_2) = 0. \quad (2.4)$$