THE REFLECTION OF LIGHT FROM GLASS WITH A THIN METALLIC FILM

ANTONÍN VALIČEK
The Institute of Physics of Masaryk University, Brno

The author deduces the correct formula for the reflection of light on a thin metallic film by the generalizing of the well known formula for the reflection of light on a thin transparent film deposited on glass. The deduced formula is in accordance with a simple consideration about the energy relationship in reflected light. In the second part the optical coherence between the reflection of light from two thin transparent films and the reflection from a thin metallic film is discussed.

I. THE GENERALIZING OF THE FORMULAE FOR THE REFLECTION OF LIGHT ON A THIN METALLIC FILM DEPOSITED ON GLASS

The problem of the reflection of light from a thin metallic film deposited on a transparent medium, e.g., on glass, has often been solved, especially recently the optical characteristics of the interference filters having thin semitransparent metallic films. These optical quantities are the reflectance and the transmittance of the thin metallic film.

For the sake of simplicity we limit our considerations to the normal incidence $\varphi = 0^\circ$ only. Calculating the reflection of light from a thin metallic film we usually generalize the relations deduced for the reflection of light from a thin transparent film on a glass surface. For the reflection of light from glass with a thin transparent film the relation was deduced

$$ r_{\text{glass}} = \frac{r' + r'' e^{-i\varphi}}{1 + r' r'' e^{-i\varphi}}, \quad (1) $$

where $r$ is the resultant amplitude, $\varphi$ the resultant phase.

The light is incident from the optical medium of refractive index $n_0$ (we usually consider air as having refractive index $n_0 = 1$), which is adjacent to the film being investigated, normal to the thin film with the refractive index $n_1$ and thickness $d_1$; the thin film is deposited on the glass of refractive index $n$. Fresnel’s amplitudes are as follows:

For the reflected light from the boundary between the adjacent medium and the film

$$ r' = \frac{n_1 - n_0}{n_1 + n_0}, \quad (2) $$

from the boundary between film and glass

$$ r'' = \frac{n - n_1}{n + n_1}. \quad (3) $$

The path difference for light of wave-length $\lambda$ is given by the formula

$$ x = 2n_1 d_1 $$

and the phase difference

$$ x = \frac{2\pi}{\lambda} 2n_1 d_1. \quad (4) $$
The Reflection of Light from Glass with a Thin Metallic Film

The previous relations, valid for the reflection of light from a thin transparent film on glass surface, are generalized for a thin metallic film in the following way; the refractive index of the transparent film \( n_1 \) is substituted by the refractive index of thin metallic film in its complex form

\[
 n_1 = n_1 - i n_1 k_1 .
\]  

where \( n_1 \) is the refractive index, \( k_1 \) the absorption index of the metallic film [1].

Fresnel's amplitude of the reflected light on the boundary between the adjacent medium and thin metallic film is generalized as follows. In Equ.(2) the complex refractive index of thin metallic film \( n_1 \) is substituted for the refractive index of the film \( n_1 \). We obtain

\[
 r'e^{-\beta'} = \frac{n_1 - n_0}{n_1 + n_0} = \frac{ni - no - im_1 k_1}{n_1 + n_0 - im_1 k_1} = \frac{n_1^2 + n_1^2 k_1^2 - i2n_0 m_1 k_1}{(n_1 + n_0)^2 + n_1^2 k_1^2} .
\]  

(6)

The angle \( \beta' \) is indicated as the phase shift in consequence of the reflection of light on the metallic medium.

Equating the real and imaginary parts we get

\[
 r' \cos \beta' = \frac{n_1^2 + n_1^2 k_1^2 - n_0^2}{(n_1 + n_0)^2 + n_1^2 k_1^2} ,
\]  

(7)

\[
 r' \sin \beta' = \frac{-2n_0 m_1 k_1}{n_1^2 + n_1^2 k_1^2 - n_0^2} ;
\]  

(7')

the phase shift \( \beta' \) is calculated from the formula

\[
 \tan \beta' = \frac{-2n_0 m_1 k_1}{n_1^2 + n_1^2 k_1^2 - n_0^2} .
\]  

(8)

For the calculation of the reflectance of the first boundary between the adjacent medium and the metallic film we multiply the expression (6) by its complex conjugate \( r'e^{-\beta'} \). Finally we obtain

\[
 r'^2 = \frac{(n_1 - n_0)^2 + n_1^2 k_1^2}{(n_1 + n_0)^2 + n_1^2 k_1^2} .
\]  

(9)

Similarly Fresnel's amplitude of light reflected on the second boundary between the metallic film and glass is calculated. The formula (3) is generalized, substituting for the complex refractive index of the metallic film \( n_1 \) the refractive index \( n_1 \) of the thin transparent film in the following way:

\[
 r''e^{-\beta''} = \frac{n - n_1}{n + n_1} = \frac{n - n_1 + i n_1 k_1}{n + n_1 - i n_1 k_1} .
\]  

(10)

As before the analogous calculations lead to the final formulae:

\[
 \tan \beta'' = \frac{2nn_1 k_1}{n^2 - n_1^2 - n_1^2 k_1^2} ,
\]  

(11)

\[
 r''^2 = \frac{(n - n_1)^2 + n_1^2 k_1^2}{(n + n_1)^2 + n_1^2 k_1^2} .
\]  

(12)

The formula (1) deduced for the reflection of light from a thin transparent medium on glass is usually generalized for the reflection of light from a thin metallic film on glass as follows: instead of Fresnel's amplitude \( r' \) the gene-