i.e. the amplitude \( B_\alpha \) decreased with increasing density velocity, the resultant relation for the density velocity was

\[
u^2 - u_{\alpha 0}^2 = \gamma_1 \frac{b_0}{M_0} \left( \gamma_2 I_0 \omega - b_0 \right) \left( t + \frac{1}{2 \omega} \sin 2 \omega t \right).
\]

Here \( u_{\alpha 0} \) is the initial cluster velocity at a time \( t = 0 \), \( \gamma_1 \) and \( \gamma_2 \) are constants, \( I_0 \) the amplitude of the exciting current. The other quantities are analogous to those in relations (2) and (3).

Relation (4) is very remarkable. It shows primarily that with increasing time the cluster velocity continually grows and from the condition

\[
\frac{\partial (u^2)}{\partial b_0} = 0
\]

an analogy of the resonance (magnetic) state with single accelerations is again obtained, i.e. it must hold for maximum velocity increment that

\[
b_0 = \frac{1}{2} \gamma_2 I_0 \omega.
\]

The induction accelerator with a radial magnetic field has maximum velocity given by relation (1) for single acceleration, and the cluster velocity continually increases according to equation (4) for continual acceleration and decreasing radial magnetic field.

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References


CIRCULAR AUTOSTABLE PATH
OF MAGNETOHYDRODYNAMIC PLASMA ACCELERATOR

The magnetic resonance states of magnetohydrodynamic accelerators of plasmatic clusters with parallel conductors, coaxial conductors or without conductors in an induction arrangement are very effective but the paths along which acceleration occurs are very long. An attempt has therefore been made to find an arrangement of an accelerator whereby a circular and, if possible, an autostable path would be possible. An autostable circular path is conceived as a path where the centrifugal force is balanced by a spontaneously produced centric magnetohydrodynamic force.

*) Praha 6, Technická 1902.
Figure 1 shows a model which was subjected to investigation. A very thin plasmatic cylindrical cover, through which a current flows in the axial direction, is rotated in a radial magnetic field. The currents in the azimuthal and radial directions are balanced by a stabilizing radial electric field and by the time change of the axial magnetic field. In this way it is possible to consider only circumferential velocity. If the equation for momentum and the generalized Ohm's law are used, the following basic equations, describing the motion of a cylindrical cover of the plasma, are obtained:

$$\frac{ds}{dt} = \sigma (E_z - u_\phi B_r) B_r - \eta u_\phi,$$

$$\frac{u_\phi^2}{r_0} = \mu_0 \frac{r_0}{2} \sigma^2 (E_z - u_\phi B_r)^2.$$

Here $\sigma$ is the specific electrical conductivity of the plasma, $E_z$ the intensity of the axial electric field and $\eta$ a coefficient which represents the dissipative losses in the acceleration process, the quantity $\mu_0$ is the permeability of vacuum. For the sake of simplicity $s, \sigma$ and $\eta$ can be regarded as constants.

It is interesting that under the above assumptions the system of equations (1) provides an autostable path, i.e. a path of constant radius $r_0$ for the whole period of acceleration [1], for only one case. It is assumed that

$$B_r = \text{const.}, \quad E_z = \delta_z u_\phi,$$

i.e. that for constant radial magnetic field the electric axial field grows in direct proportion to the velocity $u_\phi$ according to the constant $\delta_z$.

For conditions (2), Eqs. (1) give

$$u_\phi = u_{\phi 0} \exp \left\{ \frac{1}{s} \left[ \sigma (\delta_z - B_r) B_r - \eta \right] t \right\}$$

and

$$r_0 = \left( \frac{2s}{\mu_0 \sigma^2} \right)^{1/2} \frac{1}{\delta_z - B_r}.$$

This means that on an autostable path of radius $r_0$ according to (4) the velocity $u_\phi$ increases continually according to (3). The increase in circumferential velocity is limited by the technical possibilities or by relativistic effects.