The paper solves the relativistic equations of motion of a single-nucleon model of the atomic nucleus. The nucleon moves in a central field which is given by a mixture of scalar, pseudo-vector and pseudo-scalar coupling. A comparison between the results and experiment and between the results and the non-relativistic theory of the shell model shows that the relativistic theory offers practically as much as the non-relativistic theory, although it differs both in the formal and physical aspects.

INTRODUCTION

The present paper is based on [1] in which the possibility of the existence of a coupled state of a nucleon in a meson field is investigated. Here we start out from the results of [1] and assume that it is possible to construct a single-nucleon "relativistic" model of a nucleus by means of a potential well. The potential well is taken to be a sort of averaged $\pi$-meson field formed by nucleons in the nucleus.

It is known from the theory of nucleon and $\pi$-meson interaction that the coupling between nucleons and $\pi$-mesons is pseudovector if the nucleons are described as Dirac particles. The interaction constant of this coupling is $f^2 = 0.08$. In the non-relativistic approximation there is no substantial difference between pseudo-vector and pseudo-scalar coupling. Practically the same results are obtained by the theory working with pseudo-scalar interaction if the interaction constant $g^2 = 15$ is used.

It was shown in [1] that pseudo-scalar coupling belongs to those types of couplings which do not permit a bound state of a nucleon in a meson field. Neither pseudo-vector coupling nor pseudo-scalar coupling give terms in the equation of motion of a nucleon which would be analogous to the well-known spin-orbit interaction. The only coupling which gives spin-orbit terms is scalar interaction, which, however, is usually omitted in mesodynamics. With nuclear potentials, which very rapidly decrease to zero at the edge of the atomic nucleus and which are almost constant inside it, pseudo-vector coupling acts rather as a potential barrier than as an attractive potential. For these reasons the resultant interaction acting in the averaged field on the nucleon is taken as a mixture of pseudo-vector, pseudo-scalar and scalar coupling. This choice is also borne out by the results of paper [2], in which it is shown that, when describing a nucleon by anomalous equations, the mixture of these three couplings is one of the simplest cases possible.

1. SOLUTION OF EQUATION OF MOTION OF NUCLEON IN POTENTIAL FIELD

The initial equation of motion of a nucleon is written in the form

\[ \gamma \vec{p} + \gamma_0 E - mc^2 + \mu \vec{p} \gamma_s(\vec{p} \gamma V) - \varepsilon \gamma \gamma V + \lambda V^2 \psi = 0, \]

which is a combination of Eqs. (16) and (17) from [2]. Here $\gamma_0, \gamma_\gamma = (\gamma_1, \gamma_2, \gamma_3), \gamma_s$ are Dirac matrices, $\vec{p} = -i \hbar \text{grad}$, $V$ pseudo-scalar potential, $\mu, \varepsilon, \lambda$ interaction constants.
of pseudo-vector, pseudo-scalar and scalar coupling. We introduce the dimensionless quantities $\mu^*, v^*, \lambda^*$; $\mu = \mu^*/mc, \lambda = \lambda^*/mc^2, v = v^*$. The constant $|\mu^*| = |f|/m_m$, for $\lambda^* = v^* = 0$ and $|v^*| = |g|$, for $\mu^* = \lambda^* = 0$; where $m$ is the nucleon mass and $m_m$ the $\pi$-meson mass.

Assuming that the potential $V$ is spherically symmetrical, Eq. (1.1) can be transformed to the radial form (see [1], Eq. 13). Then

$$- (E - mc^2 + \lambda V^2) R_1 + \hbar c \left( R'_2 + \frac{1 + 2}{r} R_2 \right) + \mu \hbar \frac{dV}{dr} R_3 + v V R_4 = 0,$$

$$- (E - mc^2 + \lambda V^2) R_2 + \hbar c \left( R'_3 - \frac{1}{r} R_3 \right) - \mu \hbar \frac{dV}{dr} R_4 - v V R_3 = 0,$$

$$- (E - mc^2 + \lambda V^2) R_3 + \hbar c \left( R'_4 - \frac{1}{r} R_4 \right) + \mu \hbar \frac{dV}{dr} R_1 + v V R_2 = 0,$$

$$- (E - mc^2 + \lambda V^2) R_4 + \hbar c \left( R'_3 + \frac{1 + 2}{r} R_3 \right) - \mu \hbar \frac{dV}{dr} R_2 - v V R_1 = 0,$$

where $R_i (i = 1, 2, 3, 4)$ are the radial components of the wave function $\psi$, which has the form

$$\psi = \begin{pmatrix} -i(l - m + 1) R_3 Y^{m+1}_{l+1} (\theta, \phi) + i(l + m + 1) R_1 Y^m_l (\theta, \phi) \\ -i R_3 Y^{m+1}_{l+1} (\theta, \phi) - i R_1 Y^m_l (\theta, \phi) \\ -(l + m + 1) R_4 Y^m_l (\theta, \phi) + (l - m + 1) R_2 Y^{m+1}_{l+1} (\theta, \phi) \\ R_4 Y^{m+1}_{l+1} (\theta, \phi) + R_2 Y^m_l (\theta, \phi) \end{pmatrix}.$$  

This wave function is the eigenfunction corresponding to the total momentum with quantum number $j = l + 1/2$. A solution to the equations in the form of (1.2) does exist although it was shown in [1] that it does not. The incorrect assertion in [1] was based on an incomplete solution of the given system.

The system of equations for radial functions (1.2) can be relatively easily solved if the potential $V$ is chosen in the form of a square well, which is often used in the shell model of the atomic nucleus. In order to satisfy the conditions of continuity of the wave function on the boundary, we make the walls of the potential well slope very slightly and put

$$V = -V_0 \quad \text{for} \quad r \leq a;$$

$$V = -V_0 \frac{r_0-r}{r_0-a} \quad \text{for} \quad a < r < r_0; \quad r_0 - a \ll r_0;$$

$$V = 0 \quad \text{for} \quad r \geq r_0.$$

1) $l$ is not a good quantum number; here it has merely a mathematical significance.