INCIDENCE OF ELECTROMAGNETIC WAVES ON A PLASMA-VACUUM BOUNDARY

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An electromagnetic wave polarized in a plane normal to the magnetostatic field, parallel to the plasma boundary, falls from a vacuum on a half-space filled with magnetoactive plasma and bounded by a wall. The coefficient of wave reflection is computed for plasmas with \( \omega_{10}^2/\omega_0^2 \ll 1 \).

1. INTRODUCTION

The object of the study is to determine the coefficient of reflection of an electromagnetic wave polarized in the plane normal to the magnetostatic field and falling from a vacuum perpendicularly on the plasma boundary parallel to the magnetostatic field. Some of the aspects of the propagation of such a wave in case the plasma is held by an external magnetostatic field have been analyzed in [1] without consideration having been given to the magnetic field inhomogeneity. As indicated in [2], additional resonance mechanisms of the particles-wave interaction may play a role as a result of the magnetic field inhomogeneity. Such mechanisms may conveniently be described by representing the plasma boundary as a wall elastically reflecting the electrons. A wall of this sort may be considered an idealized boundary between the plasma and a strong magnetic field, or a limit case of the potential barrier formed e.g. by a high-frequency field. Having the electric vector polarized along the magnetostatic field [2], the resonance interaction is caused by the field gradient near the boundary. We can, therefore, expect a greater change in the coefficient of reflection in the case of polarization perpendicular to the magnetostatic field, as is also confirmed by the result of our paper.

2. BASIC EQUATIONS

A homogeneous plasma fills the half-space \( x > 0 \). It is bounded in the plane \( x = 0 \) by a wall on which elastic reflection of electrons takes place. The external magnetostatic field \( \mathbf{H} \) is parallel to the \( z \)-axis. An electromagnetic wave \( \mathbf{E} = \{0, E_y(x) e^{-i\omega t}, 0\} \) falls from a vacuum on the plasma boundary. The electromagnetic field in the vacuum is a sum of the incident and the reflected waves:

\[
E_y = E_{y0} e^{ikx-\omega t} + E_{y1} e^{-ikx-\omega t}.
\]

We shall discuss the case of the wave penetrating into the plasma, i.e. in the magneto-hydrodynamic approximation in regions \( \omega^2 < \omega_0^2 + \omega_s^2 \) and \( \omega^2 > \frac{1}{2} [\omega_0^2 + 2\omega_s^2 + \sqrt{(\omega_0^4 + 4\omega_0^2\omega_s^2)}] \) where \( \omega_0 = eH/mc \) and \( \omega_s = 4\pi e^2N/m. \)
The solution starts from Vlasov's equation and Maxwell's equations. The ions are considered immobile background. The distribution function

\[ f = f_0 + f_1 \]

where \( f_0 \) is the Maxwell distribution function and \( f_1 \) is a small disturbance of the equilibrium state satisfying the equation

\[ \frac{\partial f_1}{\partial t} + v_x \frac{\partial f_1}{\partial x} - \omega_H \left( v_y \frac{\partial f_1}{\partial v_x} - v_x \frac{\partial f_1}{\partial v_y} \right) = \frac{e}{m} \left[ E_x(x, t) \frac{\partial f_0}{\partial v_x} + E_y(x, t) \frac{\partial f_0}{\partial v_y} \right] \]

The solution of this equation \([3]\) is taken in the form

\[ f_1 = \frac{e}{m} \int_{-\infty}^{t'} \left\{ E_x(x(t' - t'), t') \frac{\partial f_0}{\partial v_x} + E_y(x(t' - t'), t') \frac{\partial f_0}{\partial v_y} \right\} dt' \]

where \( x(t' - t') \) is the position in time \( t' \) of a particle with coordinate \( x \) and velocity \( \mathbf{v} \) in time \( t \). The quantity is computed without the effect of the wave field. The electric field satisfies Maxwell's equations

\[ \frac{\partial E_x}{\partial x} = -4\pi e \int f_1 \, d\mathbf{v} \]

\[ \frac{\partial^2 E_x}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} = -\frac{4\pi e}{c^2} \int v_y f_1 \, d\mathbf{v} . \]

A solution of equations (2), (3) will be sought in the form

\[ E_x(x) = E_{x2}[e^{hx} + u(x)] \]

\[ E_y(x) = E_{y2}[e^{hx} + w(x)] , \]

where \( u(x) \) and \( w(x) \) are new functions approaching zero with increasing \( x \). The constants \( h \), \( E_{x2} \), \( E_{y2} \) are determined from equations describing the propagation in an unconfined plasma

\[ i a_0 E_{x2} = \frac{2a_0}{\pi a_H} \int_0^{2\pi} \sin^2 \delta \sin^2 \theta \left\{ \int_0^\infty x^2 e^{-x^2} dx \right\} \int_0^\infty \cos (\delta - \tau) E_{x2} \exp \left[ i \Omega \tau + i a_0 x \left( \sin (\delta - \tau) - \sin \theta \right) + E_{x2} \sin (\delta - \tau) \exp \left[ i \Omega \tau + i a_0 x \left( \sin (\delta - \tau) - \sin \theta \right) \right] \right\} \]

\[ E_{y2} \left( \frac{\omega^2}{c^2} - h^2 \right) = -\frac{2i\Omega a_0}{\pi} \int_0^{2\pi} \sin \delta \, d\delta \left\{ \int_0^\infty x^2 e^{-x^2} dx \right\} \int_0^\infty \cos (\delta - \tau) \left\{ \left[ E_{x2} \cos (\delta - \tau) + E_{y2} \sin (\delta - \tau) \exp \left[ i \Omega \tau + i a_0 x \left( \sin (\delta - \tau) - \sin \theta \right) \right] \right\} \right\} . \]