THE USE OF ACOUSTIC AXES IN DECIDING BETWEEN THE VOIGT AND LAVAL-RAMAN THEORIES OF THE ELASTIC PROPERTIES OF CRYSTALS

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A method is derived for finding the crystal directions that are acoustic axes, independently of the values of the elastic constants and of the assumption of the validity of Voigt’s theory of the elastic properties of crystals. If, on the other hand, the Laval-Raman theory is satisfactory for the given crystal, the found directions lose the properties of acoustic axes. Measurement of the phase velocity of elastic waves in the found directions makes it possible to decide for each crystal whether these directions are really acoustic axes and thus to determine which of the two theories is valid.

1. INTRODUCTION

When deciding on the validity of the Voigt or Laval-Raman theory of the elastic properties of crystals by means of measuring the phase velocities of elastic waves it is possible to use acoustic axes. These are the directions in the crystals in which both shear waves of the three so-called isonormal waves, which can propagate through the crystal in each direction, have the same value of the phase velocity1).

A necessary and sufficient condition for a direction in the crystal to be an acoustic axis is the fulfilment of the following condition given in [2], p. 119 with reference to paper [4]:

\[ 27|A|^2 + 4(A_C)^3 + 4A_C^2|A| - A_C^2(A_C)^3 - 18A_C|A|A_C = 0 \]

The starting point for deriving this condition is the Christoffel theory of plane elastic waves in an unlimited medium [5]. The quantities \( A_C \), \( \lambda_C \) and \( |A| \) here are defined by

\[ A_C = A_{11} + A_{22} + A_{33} \]
\[ \lambda_C = A_{11}A_{22} + A_{22}A_{33} + A_{33}A_{11} - A_{13}^2 - A_{23}^2 - A_{12}^2 \]
\[ |A| = A_{11}A_{22}A_{33} + 2A_{12}A_{23}A_{33} - A_{11}A_{23}^2 - A_{22}A_{13}^2 - A_{33}A_{12}^2 \]

where

\[ A_{ik} = \frac{1}{q} C_{ijkl} s_j s_l \]

Here \( q \) denotes the density of the crystal in question, \( C_{ijkl} \) the tensor components of the elastic moduli and \( s_0, s_j \) the direction cosines of the wave normal \((i, j, k, l = 1, 2, 3)\) which will be the acoustic axis if condition (1) is fulfilled.

Condition (1) is valid for both theories of elastic properties if we always substitute suitable components \( C_{ijkl} \) corresponding to the theory considered and the given crystal. As a consequence of the general difference in the form of the tensor of elastic moduli in the two theories (cf. e.g. [8], pp. 296–301 and [6]) we should arrive in each of the two possible cases at a different set of acoustic axes.

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1) If the direction of propagation is the acoustic axis, the two mentioned waves are really purely shear waves [2], pp. 108–109.
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axes of the same crystal. If we find a direction in the crystal that satisfies condition (1) only within the framework of one of the two theories, the measurement of the same or different phase velocities of both shear or quasi-shear waves in this direction makes it possible to decide directly in favour of one or the other.

When proposing the above experiment it is necessary to ensure that the direction of the acoustic axis in the crystal does not depend on the values of its elastic moduli $C_{ijkt}$. This is because if we are deciding between the two theories, we primarily do not know which of the two tensors of elastic moduli is satisfactory here, and this prevents us from determining at least some of the values of the elastic moduli beforehand. In practice, this means that our requirement that the found direction always be the acoustic axis for one of the theories considered, must be satisfied for an arbitrary possible choice of elastic moduli.

Because the transition from the Laval to the Voigt tensor of elastic moduli can be formally imagined by the replacement of always four generally different moduli by a single Voigt modulus, all the acoustic axes considered by us are calculated using the Laval tensor of elastic moduli of the crystal in question by the sub-set of the set of acoustic axes corresponding to the Voigt tensor.

2. CALCULATION OF ACOUSTIC AXES OF CRYSTALS

In the actual calculation of the directions of the sought acoustic axes we first substitute from relations (2) and (3) into condition (1). After rearrangement the left-hand side of condition (i) contains the sum of terms where each consists of a combination of positive powers of the elastic moduli, altogether always of the sixth degree, multiplied by a coefficient that depends only on the direction cosines of the wave normal.

The moduli used, $C_{ijkt}$, characterizing the elastic properties of the crystals are always chosen so that within the framework of phenomenology there is no logical reason why in the crystal in question they could not attain very different values independently of one another. Our requirement of invariance of the direction of the acoustic axis in the crystal then means that condition (1) must also be fulfilled for all their possible values. It thus follows that the coefficients at the different powers of the elastic moduli in condition (1) must all be zero.

By solving the equations obtained in this way, on the assumption that in definition (3) the quantities $C_{ijkl}$ have been successively replaced by the components of the Voigt and Laval tensors of elastic moduli of the crystal in question, we obtain the sets of acoustic axes corresponding to both theories and the given crystal. By eliminating the directions that belong to both sets, we finally obtain the directions that are acoustic axes only if Voigt's theory is valid for the given crystal. The proposed experiment can be performed for each of the acoustic axes determined in this way.

3. EXAMPLE

As an example we found the acoustic axes of the required properties for crystals of crystal class $m\overline{3}$. In agreement with the form of the tensor of elastic moduli of this symmetry in Voigt's theory ([8], p. 301), we substituted definitions (2) and (3) into condition (1). The coefficients found at the

2) If we find by measurement that the corresponding velocities of the two waves in the crystal are different, their common wave normal is not in reality an acoustic axis and the two waves need not necessarily be purely shear.

3) This procedure, necessary for solving our problem, also makes it possible to avoid finding the acoustic axes directly by means of condition (1) which is somewhat complicated for this purpose.