ON THE TRANSIENT ELECTRICAL RESPONSE OF A PIEZOELECTRIC TRANSDUCER DUE TO SHOCK-LOADED STRESS

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The methods of transform calculus are made use of in finding the electrical response in a piezoelectric transducer due to shock-loaded stress. The response is found to be transient in character to a first order of approximation.

1. INTRODUCTION

The sphere of applications of piezoelectric materials is fast widening its radius, particularly in regard to transducers which are used to generate and to detect ultrasonic waves. The piezoelectric materials used for ultrasonic purposes can emit mechanical responses due to electrical excitations and vice-versa. Mason [11] has carried out this analysis by pursuing the principles of circuit theory. The methods of continuous media have been followed by Redwood [2, 3], and some papers have been contributed by Sinha [4-7], Giri [8], Das [9], Roy [10]. The author here attempts to find out the electrical response emitted by a piezoelectric transducer due to a transient mechanical signal. The analysis proceeds on the assumptions in relation to shock-loaded stress in quartz made by Graham, Neilson, Bendick [11].

2. FUNDAMENTAL EQUATIONS, BOUNDARY CONDITIONS

We consider a piezoelectric transducer in the form of a plate transducer and the current produced during the propagation of a stress-wave along the x-axis of the plate which is taken to be the thickness-direction of the plate. To obtain the electrical response we have to couple the equations connecting the two fields — mechanical and electrical. The displacement current $i$ due to the stress wave, as in (11), is given by

\[ i = A' \frac{dD}{dt}, \]

where $A'$ is the electroded area, $D$ the electrical displacement.

Also from [11],

\[ D = \frac{1}{x} \int_{0}^{x} P(x) \, dx, \]

where $x$ is the thickness of the plate and $P$ is the electric polarization.

In accordance with [11], $P(x) = f \sigma(x)$, where $f$ is a constant and $\sigma$ the mechanical tension in the direction of the thickness of the plate.
We get from (1)
\[ i = A' \frac{d}{x} \int_{0}^{x} \sigma(x, t) \, dx \]
(2)
\[ i = A' \frac{d}{x} \int_{0}^{x} \sigma(x, p) \, dx \, . \]

The constitutive relation between stress and strain is given by
\[ \sigma = cs - eE \]
(3)
\[ = cs - \frac{e}{e} (D - P) \therefore D = P + eE, \]

where \( c \) is the elastic constant, \( e \) the piezoelectric modulus and \( e \) the dielectric permittivity.

Therefore,
\[ \frac{\delta \sigma}{\delta x} = c \frac{\delta s}{\delta x} - e \frac{\delta (D - P)}{\delta x} = \frac{\delta D}{\delta x} \, . \]
(4)

From Gauss' law, since there is no free charge inside the transducer,
\[ \text{div} \, \mathbf{D} = 0 . \]
(5)

Since propagation of the stress wave is assumed to be plane in character, differentiation with respect to \( y \) and \( z \) is zero; i.e.
\[ \frac{\delta D_y}{\delta y} = \frac{\delta D_z}{\delta z} = 0 . \]

Therefore, from equation (5),
\[ \frac{\delta D_x}{\delta x} = 0 . \]

Equation (4) becomes
\[ \frac{\delta \sigma}{\delta x} = c \frac{\delta s}{\delta x} + e \frac{\delta P}{\delta x} \, . \]
\[ \therefore p \frac{\delta^2 \xi}{\delta t^2} = c \frac{\delta^2 \xi}{\delta x^2} + \frac{pef}{e} \frac{\delta^2 \xi}{\delta t^2} \, . \]

where \( \xi \) is the displacement along the \( x \)-axis. Taking Laplace's transform, we have
(6)
\[ \frac{\delta^2 \xi}{\delta x^2} - \frac{p}{c} \left( 1 - \frac{ef}{e} \right) p^2 \xi = 0 . \]