ON THE DETERMINATION OF THE STATISTICAL PROPERTIES OF LIGHT FROM PHOTOELECTRIC MEASUREMENTS

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A relation is derived between the probability distribution of the integrated intensity of light incident on a photoelectric detector and that of the emission of $n$ photoelectrons in a time interval $(t, t + T)$ from the properties of a quantized electromagnetic field. Both the direct and the inverse relations are derived, thereby enabling the statistical properties of light to be determined from photoelectric measurements. An explicit expression is given for the coefficients for calculating the moments $\langle W^k \rangle$ by means of the moments $\langle n^j \rangle (j = 1, 2, \ldots, k)$. The correctness of the operations in which generalized functions occur is confirmed by some direct calculations.

1. INTRODUCTION

The significance of photoelectric detectors has been increasing of late. Since their response is proportional to the intensity of incident light (to the square of the amplitude of light) (more exactly, to the local mean time value of the intensity), one speaks of quadratic detectors. These detectors are of basic importance in the theory of the coherence of the electromagnetic field, since if we place $N$ such detectors into $N$ different points in space and if we lead the outputs of these detectors to the correlator we can measure the correlation (coherence) effects of the $2N$-th order in the given field [1]. Such correlation measurements provide information on the energy distribution in the radiation spectrum [2], the angular dimensions of stars [3], and the state of polarization [4] and also enable the phase of physically important functions (e.g. the scattering amplitudes) [5] to be determined. Although in such experiments the properties of photoelectrons are studied rather than of photons, they reflect the basic properties of photons.

Let the probability distribution of the emission of $n$ photoelectrons in a time interval $(t, t + T)$ be $p(n, T, t) = p(n)$ and let the probability distribution of the quantity $W(T, t) = \int_t^{t+T} I(t') \, dt'$, where $I(t)$ is the intensity of linearly polarized light incident on the photodetector (for simplicity we assume that the sensitivity of the photocathode is $\alpha = 1$), be $P(W)$. The quantity $W$ can be regarded as a classical characteristic of the field, the quantity $n$ represents the quantum characteristic of the field. Then, on the basis of such a semi-classical description, in which the field is described classically, but its interaction is described quantum-wise, one can derive the following relation [6]

$$
p(n) = \int_0^\infty P(W) \frac{W^n}{n!} \exp(-W) \, dW,
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i.e. \( p(n) \) is the integral representation of \( P(W) \) with Poisson kernel. Relation (1) can be inverted in the form \([7, 8]\)

\[
(2) \quad P(W) = \exp(W) \sum_{n=0}^{\infty} (-1)^n p(n) \delta^{(n)}(W).
\]

If we confine ourselves to calculating the moments (correlation functions) of the distribution \( P(W) \), we can regard (2) as a generalized function. In general, however, for all functions \( f(W) \) of the given class \( \int_{-\infty}^{\infty} P(W, f(W, dW) \) will not be convergent (e.g. for \( f(W) = \exp W, \) \( p(n) = (n) n^a(n) \) valid for thermal radiation in thermal equilibrium, where \( \langle n \rangle \) is the mean number of particles, see [9]). Relation (2) permits \( P(W) \) to be determined from a knowledge of \( p(n) \), i.e. the statistical properties of light to be determined from photoelectric measurements.

Let us now pay attention to some typical examples of the probability distributions \( p(n) \) and \( P(W) \). For example, for classical particles \( p(n) = (\langle n \rangle^a n) \exp(-\langle n \rangle) \) (Poisson distribution) and after substituting into (2) we obtain \( P(W) = \delta(W - \langle n \rangle) \). This is the case of a completely stabilized laser source. Another interesting case is that of thermal radiation. As has already been said, in this case \( p(n) = (\langle n \rangle^a n) \exp(-\langle n \rangle) \) (Bose-Einstein distribution) and after substituting into (2) and using the integral representation \( \delta^{(n)}(W) = 1/2\pi \int_{-\infty}^{\infty} (ix)^n e^{-ixW} \) we obtain \( P(W) = (\langle n \rangle - i \exp[-W/(\langle n \rangle)]. \) In this case (2) gives an ordinary function.

Relation (1) was derived from the properties of a quantized electromagnetic field in [10] and relation (2) in [8]. Here we give a somewhat different method of deriving relation (1) from the properties of a quantized electromagnetic field than the method given in [10]. Our method throws greater light on the relation between moments of the distributions \( p(n) \) and \( P(W) \) and on the relation between the quantum mechanical moments calculated by means of commutation relations for field operators. We refine and slightly change the method of deriving relation (2) from the properties of a quantized electromagnetic field given in [8]. We give the explicit expression of the coefficients for calculating the moments \( \langle W^k \rangle \) by means of the moments \( \langle n^j \rangle \) \((j = 1, 2, \ldots, k)\) (relation (45)) which are simpler than the coefficients for expressing the moments \( \langle n^k \rangle \) by means of \( \langle W^j \rangle \) \((j = 1, 2, \ldots, k)\) and we verify by direct calculations the correctness of the operations in which generalized functions occur.

2. BASIC FORMALISM OF THE QUANTUM THEORY OF AN ELECTROMAGNETIC FIELD

Before showing that relations (1) and (2) are the natural consequence of defining the operator of the number of particles in the quantum field theory, we shall pay attention to some of the relations from the quantum theory of an electromagnetic field.

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1) We define \( \int_{-\infty}^{\infty} \delta(W) dW = 1. \)