The linear theory of the two-beam instability of weak gyrating beams is investigated. Maximum growth rates of instabilities, instability limits and frequencies of unstable waves are numerically determined. The aperiodic instability with \( k_z v_z = \omega_c \) \((v_z \text{ is the velocity of the beam along the magnetic field, } \omega_c \text{ is the cyclotron frequency and } k_z \text{ is the longitudinal wave number})\) and the instability with \( v - \omega_c/2 \) and with \( k_z v_z = \omega_c/2 \) possess greatest growth rates. Dependence of growth rates on longitudinal and transversal components of the wave vector is investigated in detail.

1. INTRODUCTION

The interaction of two counter-streaming electron beams, which has been studied in a number of experiments \([1-3]\), leads to the excitation of mostly electrostatic waves. In the presence of an external magnetic field the spectrum of waves excited by weak beams was observed to consist of integer multiples of the half of the cyclotron frequency \([1, 2]\). Also the values of the longitudinal wave vectors were close to multiples of the quantity \( \omega_c/2v_z \). These results are in a good agreement with simple theory of the excitation of electrostatic waves by counter-streaming electron beams with a finite transversal energy of particles \([2]\). However, the growth rates of unstable waves have been investigated only in some special cases.. E.g. in \([4]\) only the excitation of waves, when both beams interact by means of the anomalous Doppler mechanism at the same harmonic cyclotron frequency was discussed. The growth rates derived in \([2, 5]\) are valid only in some special cases (small transversal wavelengths \([5]\), small transversal velocity of the beams \([2]\)).

In this paper the electrostatic instabilities of two homogeneous counter-streaming electron beams in the magnetostatic field are studied in detail. The beam density is supposed to be small so that the condition \( \omega_o \ll \omega_c \) \((\omega_o \text{ is the plasma frequency})\) is fulfilled. Under the assumption that the thermal spread of the beam velocities both transversal and longitudinal is negligible, the dispersion equation has been investigated numerically; on the basis of this investigation we have derived the limits of the instability in the space of wave vectors, dependence of the growth rate on the longitudinal and transversal wave numbers and the maximum growth rate for particular modes.
2. ANALYSIS OF DISPERSION EQUATION

The dispersion equation of electrostatic waves for the system of two homogeneous beams with the distribution function

\[ f_0 = \frac{n_0}{4\pi v_{z0}} \delta(v_z - v_{z0}) \left[ \delta(v_x - v_{z0}) + \delta(v_x + v_{z0}) \right] \]

is of the following form

\[ \frac{k_z^2}{k^2} = \frac{n_0}{2} \sum_{n=-\infty}^{+\infty} \left\{ \frac{J_n(k_z q)}{k_z v_{z0}} \left\{ \frac{1}{(\omega - n\omega_c - k_z v_{z0})^2 + (\omega - n\omega_c + k_z v_{z0})^2} \right\} + \frac{k_z n J_n J_n' \left\{ \frac{1}{(\omega - n\omega_c - k_z v_{z0})^2 + (\omega - n\omega_c + k_z v_{z0})^2} \right\}}{k_z^2 v_{z0}} \right\} + \frac{1}{\omega - n\omega_c + k_z v_{z0}} \].

In deriving eq. (2) the coordinate system has been chosen with the \(z\)-axis parallel to the external homogeneous magnetic field. The Larmor radius of electrons is \(r_0 = v_{z0}/\omega_c\), \(k_z, k_x\) are components of the wave vector, \(J_n\) is the Bessel function. Further, the electron beams have been supposed to be neutralized by the immobile ion background. The ion terms in the dispersion equation can, however, be neglected.

In the region of small densities \((\omega_0^2 \ll \omega_c^2)\) the frequencies satisfying approximately some of the resonance conditions

\[ \omega \approx n\omega_c \pm k_z v_{z0} \]

can only be solutions of the dispersion equation (2). Only those waves can be unstable for which the resonance conditions for both beams together are fulfilled, i.e. the frequency and the wave number are given by

\[ \omega \approx \frac{n + l}{2} \omega_c, \quad k_z v_{z0} \approx \frac{l - n}{2} \omega_c, \quad n, l = -2, -1, 0, 1, 2, \ldots \]

Because of the symmetry of the problem only positive frequencies \((\omega \geq 0)\) and positive wave vectors \((k_z > 0)\) are sufficient to be considered.

The waves with \(k_z v_{z0} \ll \omega_c\) \((l = n)\) are unstable only at higher beam densities. In the case when both beams interact with waves by means of the Čerenkov effect \((l = n = 0)\) the aperiodic instability with the growth rate

\[ \gamma = \sqrt{\left\{ k_z^2 v_{z0}^2 + \frac{\omega_0^2 J_0^2 k_z^2}{2k_z^2} \right\} - \sqrt{\left\{ 2k_z^2 v_{z0}^2 \frac{\omega_0^2 J_0^2}{k_z^2} + \frac{\omega_0^2 J_0^2 k_z^2}{4k_z^4} \right\}}} \]

arises. In an unbounded medium the increment (5) has the maximum \(\omega_0/2\sqrt{2}\) at \(k_z = 0\) and \(k_z = \sqrt{(3)} \omega_0/2v_{z0} \sqrt{2}\). In a bounded medium, the beams having the