THE EFFECT OF ION MOTION ON THE REFLECTION OF AN ELECTROMAGNETIC WAVE FROM PLASMA

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An electromagnetic wave with electric vector parallel to the plasma boundary falls from a vacuum on a half-space filled with magnetoactive plasma and bounded by a particle-reflecting wall. A comparison is made of the effect of thermal motion of electrons and ions on the boundary, on the coefficient of reflection and on the absorbed energy.

1. INTRODUCTION

The paper determines the effect of ion motion on the propagation of an electromagnetic wave for cases analyzed in [1] and [2] in which ions were considered immobile background. The energy absorbed by the plasma in consequence of additional resonance mechanisms [1] is computed for the case when the wave does not penetrate into the plasma. The coefficient of wave reflection is determined for the wave propagating in the plasma.

2. SOLUTION

A homogeneous plasma fills half-space \( x > 0 \) and in the plane \( x = 0 \) is bounded by a wall that elastically reflects particles [1], [3, p. 116]. The magnetostatic field \( H_0 \) is parallel to the \( z \) axis. The electromagnetic wave \( E_0 \exp \left[ i(kx - \omega t) \right] \) falls from a vacuum on the plasma boundary. In the first case we consider the electric vector polarized parallel to the magnetostatic field. In the second case, the electric vector is normal to the magnetostatic field. The electric vector parallel to the boundary and oriented obliquely to \( H_0 \) can be resolved vectorially and the solution thus converted to that of the above-mentioned cases of polarization. If the wave does not propagate in the plasma, it can be described by

\[
E_s(x) = E_{ia}(\exp[-h r_{H_i} + u_0(\xi)]
\]

where \( \xi = x/r_{H_i} \) and \( h \) are determined from the dispersion relation of an unbounded medium [1], [4, p. 30]. If the wave propagates through the plasma, the resolution is in the form

\[
E_s(\exp[ih \xi_0 r_{H_i} + u_0(\xi_0)]
\]

[2]. The function \( u_0(\xi_0) \to 0 \), \( \xi_0 \to \infty \). If the wave does not propagate, the expression of absorbed energy is in the form [1] up to terms of the first order inclusive in the expansion with respect to small parameters

\[
S = \frac{c}{8\pi} \left( |E_0|^2 - |E_s|^2 \right) = \frac{c}{2\pi} |E_0|^2 \frac{kh}{k^2 + h^2} \text{Im} \left\{ \frac{u_0(0) + u_0'(0)}{h r_{H_i}} + \frac{u_0(0) + u_0'(0)}{h r_{H_i}} \right\}
\]
β_e (β_i) is the ratio of electron (ion) pressure and magnetic field pressure. If the wave propagates, the coefficient of reflection takes the form of [2].

\[
R = \left( \frac{|E_1|^2}{|E_0|^2} \right) = \left( \frac{k - \omega_0}{k + \omega_0} \right)^2.
\]

\[
\left\{ 1 + \frac{4k\omega_0}{k^2 - \omega_0^2} \left[ \text{Re} \left( u_{ee}(0) + u_{ii}(0) \right) - \text{Im} \left( \frac{u'_{ee}(0)}{\omega_0^2 \omega_{He}} + \frac{u'_{ii}(0)}{\omega_0^2 \omega_{Hi}} \right) \right] \right\}.
\]

In (1) and (2) \(E_0, E_1\) is the amplitude of the incident and reflected wave, \(k, h\) — the wave vector in the vacuum and in the plasma, \(h_0 = \text{Re} h, r_{He}\) and \(r_{Hi}\) — the cyclotron radii of electrons and ions, \(u_{ee}(\xi_e)\) and \(u_{ei}(\xi_i)\) are the functions corresponding to electrons and ions, \(\xi_e = x/r_{He}, \kappa = \kappa\) when \(E_0 \parallel H_0\) and \(\kappa = \kappa\), when \(E_0 \perp H_0\). Similarly as in [1] and [2] these functions are the solution of a system of integral equations obtained from the linearized kinetic equation in the collisionless approximation for electrons and ions, and from Maxwell's equations. From this system we determine \(u_{ee}(0), u_{ii}(0), u'_{ee}(0), u'_{ii}(0)\) on the assumption of the smallness of the parameters \(\Omega_e \beta_e, \Omega_i \beta_i, \omega_0^2 \omega_{He}, \omega_0^2 \omega_{Hi}\) where \(\Omega_e\) and \(\Omega_i\) are \(\omega/\omega_{He}\) and \(\omega/\omega_{Hi}\), and \(\omega_{Pe}, \omega_{Pi}\) are the plasma frequencies of electrons and ions. The solution is valid for \(\Omega_{e(i)} \neq n\) where \(n\) is a natural number.

3. CONCLUSION

If \(E_0 \parallel H_0\) and the wave does not penetrate into the plasma, the first three terms \(\text{Im} u_{e(i)}(0)\) of the expansion with respect to small parameters \(h r_{He(i)}\) are equal to zero and the first two terms of expansion \(\text{Im} u_{e(i)}(0)\) give

\[
S = \frac{3c}{8\pi^{3/2}} |E_0|^2 \frac{kh}{k^2 + h^2}.
\]

\[
\left\{ \frac{\Omega_e \beta_e h r_{He}}{(1 - \Omega_e^2)^2} \text{Im} \int_0^\infty \cos \Omega_e \varphi \sin^3 \varphi \, d\varphi + \frac{\Omega_i \beta_i h r_{Hi}}{(1 - \Omega_i^2)^2} \text{Im} \int_0^\infty \cos \Omega_i \varphi \sin^3 \varphi \, d\varphi \right\}.
\]

The imaginary part of the integral equals

\[
-\frac{\pi}{\Omega_{e(i)}} \sum_{k=1}^{N_{e(i)}} \frac{k\pi}{\Omega_{e(i)}} \frac{\sin^3 k\pi}{\Omega_{e(i)}}
\]

where \(N_{e(i)} = [\Omega_{e(i)}]\). The brackets denote as usual the next lower natural number. For large \(\Omega_{e(i)}\) the sum

\[
\frac{\sum_{k=1}^{N_{e(i)}} \sin^3 k\pi}{\Omega_{e(i)}} \approx \frac{\frac{4}{3} \pi}{\Omega_{e(i)}}.
\]