AN INFLUENCE OF THE ELECTRODES ON THE FREQUENCY OF PIEZOELECTRIC BARS*)

J. Suk

Research Institute of Radiocommunication, Prague**)

An influence of the vacuum deposited electrodes on the frequency of longitudinally vibrating piezoelectric quartz bars was studied. Both theoretical considerations and measurements show a linear dependence of the frequency on the mass and elastic properties of the electrodes. The influence of the electrodes on the frequency can be eliminated for certain ratio of the length of electrodes to the length of bar \(l_1/l_0\). A discrepancy between the theoretical and experimental value of this ratio is ascribed to various values of Young's modulus of the bulk metal and a vacuum deposited layer.

1. INTRODUCTION

The resonant frequency of a longitudinally vibrating piezoelectric bar is determined by its length and elastic properties. The influence of the cross section of the bar on its frequency \([1, 2]\) may be neglected in most cases. An air-gap between the surface of major planes of the bar and the electrodes has also some influence on the frequency. This question was studied by Cady \([3]\) both for unplated and fully plated bars with equipotential surfaces. The influence on the parameters of the equivalent circuit has been studied by Mason \([4]\). The aim of this contribution is a theoretical and experimental investigation of the influence of the length and thickness of electrodes on the frequency of longitudinally vibrating piezoelectric quartz bars.

2.1. The frequency of the unplated bar

The frequency of the unloaded bar can be derived on the basis of Rayleigh's principle; according to it in a closed system performing harmonic vibrations with a constant phase, the mean kinetic energy \(\bar{E}_{\text{kin}}\) and mean potential energy \(\bar{E}_{\text{pot}}\) are equal in the course of one period. This method presumes that the distribution of the components of mechanical displacement is known \([5]\).

In the longitudinal vibrating bar the mechanical displacement proceeds in the direction of the length of the bar and the distribution of the components of mechanical displacement in a chosen coordinate system (Fig. 1) may be written in the form

\[
(1) \quad w = w_0 \sin \omega t \sin \frac{\pi y}{l_0}.
\]

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*) Devoted to Professor V. Petržilka on his sixty-fifth birthday.

**) Novodvorská 994, Praha 4, Czechoslovakia.
The mean values of the kinetic and potential energy of the bar during one period are given by the expressions

\[ \begin{align*}
0E_{\text{kin.}} &= \frac{1}{2}q_0 \tau \int_0^\tau \int \left( \frac{\partial w}{\partial t} \right)^2 dx dy dz, \\
0E_{\text{pot.}} &= \frac{1}{2} \int_0^\tau \int \left( \frac{\partial w}{\partial y} \right)^2 dx dy dz.
\end{align*} \tag{2} \]

From the equality \(0E_{\text{kin.}} = 0E_{\text{pot.}}\) we can derive the frequency \(\omega_0\) of the unplated bar

\[ \omega_0^2 = \frac{E_0 \left( \frac{\pi}{l_0} \right)^2 \int_0^{1 \tau} \sin^2 \omega t dt \int_0^{1/2l_0} \cos^2 \frac{\pi y}{l_0} dy}{q_0 \int_0^{1 \tau} \cos^2 \omega t dt \int_0^{1/2l_0} \sin^2 \frac{\pi y}{l_0} dy} = \frac{E_0 \left( \frac{\pi}{l_0} \right)^2}{q_0 \left( \frac{\pi}{l_0} \right)} \tag{3} \]

where \(E_0\) is Young's modulus in the direction of the length of the bar, \(q_0\) is the density and \(l_0\) the length of the bar.

This equation holds in the case that the cross section of the bar can be neglected in comparison with its length. Otherwise corrected expressions must be used [1, 2].

2.2. The frequency of the plated bar

If the bar is coated with vacuum deposited electrodes arranged according to Fig. 1, we assume that the distribution of the mechanical displacement is not changed. The electrodes cause a change of frequency and of the distribution of the kinetic and potential energy between the bar and electrodes. For this system it will hold: \(0E_{\text{kin.}} + 1E_{\text{kin.}} = 0E_{\text{pot.}} + 1E_{\text{pot.}}\) and for the frequency \(\omega_1\) of the plated bar we can write

\[ \omega_1^2 = \frac{E_0 \left( \frac{\pi}{l_0} \right)^2}{q_0 \left( \frac{\pi}{l_0} \right)} \left( 1 + \frac{a_0d_0}{E_0} \left( \frac{1}{\pi(l_1/l_0)} - \frac{1}{\sin \frac{\pi l_1}{l_0}} \right) \right)^{1/2} \tag{4} \]

By the integration of this expression we obtain

\[ \omega_1 = \omega_0 \left[ 1 + \frac{a_1d_1l_1}{a_0d_0l_0} \frac{E_1}{E_0} \left( 1 + \frac{1}{\pi(l_1/l_0)} - \frac{1}{\sin \frac{\pi l_1}{l_0}} \right) \right]^{1/2} \]