THE $N(s, r)/D(s, r)$ APPROACH AND THE STANDARD EQUATIONS OF THE INVERSE SCATTERING PROBLEM*)

J. WEISS

Engineering Faculty, Slovak Technical University, Bratislava

In the non-relativistic case generating only the short-range class of potentials we explain the fact that the Marchenko equations of the inverse problem formalism are a reduced form of the generalized $N/D$ equations (the $N/D$ equations extended off the mass shell [1]).

1. INTRODUCTION

The inverse problem of the theory of potential scattering attracted an increased attention in the last few years. In various problems of the inversion character it is usual, as it seems, to use the Marchenko formalism as a powerful tool.

Recently in paper [1] Petráš proposed a method which extends the validity of the conventional $N/D$ method off the mass shell. The generalized $N/D$ equations in this approach permit to derive potentials and wave functions from the usual $N/D$ input data. It turns out that if the whole scattering amplitude is given as an input, then in terms of the $N(s, r)/D(s, r)$ equations one may formulate a possible — non-standard — way of solving the inversion scattering problem. The non-relativistic $N(s, r)/D(s, r)$ approach established in [1] for the Yukawa class of potentials has the immediate relation to the Schrödinger equation. Therefore in the role the non-standard method of the inversion problem it can help then to clarify the connection between the fundamental conceptions of the current $N/D$ formalism and quantum mechanics.

In the present paper we try to show the relation between both procedures of the inverse problem: standard Marchenko’s and non-standard, based on the $N(s, r)/D(s, r)$ equations. We note that we restrict ourselves only to the non-relativistic and more frequently occurring case when the class of potentials reproduced by the inverse formalism is the Yukawa-type one. In this case, determining correctly the left-hand-cut discontinuity, the fundamental equation of the $N(s, r)/D(s, r)$ approach leads to the Fredholm integral equation which represents the Marchenko equation with the input data necessary for finding the considered short-range potentials family.

First (in Sec. 2) we want to demonstrate for a particular case of s-wave the transition from the $N_0(s, r)/D_0(s, r)$ equation towards the Marchenko equation. Then in order to show the equivalence of both methods we proceed to the derivation of the $N(s, r)/D(s, r)$ equations starting from the corresponding Marchenko equations for the higher partial waves (Sec. 3). Finally in Sec. 4 we shall obtain the integral equation for the Jost solutions using the generalized $D(s, r)$ function and discuss the possibility of including the bound states in the $N(s, r)/D(s, r)$ equations.

*) Dedicated to Professor V. Votruba on his sixtieth birthday.

The fundamental equation of the \( N(s, r)/D(s, r) \) method is the following equation

\[
N_0(s, r) = A_0(s) e^{2i\pi r} \left[ 1 + \frac{1}{2\pi} \oint_c \frac{k' - k}{s' - s} N_0(s', r) \, ds' \right].
\]

Here \( A_0(s) \) is the \( s \)-wave scattering amplitude, \( s \) and \( k \) are the energy and momentum variables respectively and the \( D_0(s, r) \) function, defined by the expression

\[
D_0(s, r) = 1 + \frac{1}{2\pi} \oint_c \frac{k' - k}{s' - s} N_0(s', r) \, ds',
\]

contains a contour integral with the contour pictured in Fig. 1. The potential \( u_0(r) \) have to be calculated from the relation

\[
u_0(r) = \frac{i}{\pi} \oint_c N_0(s, r) \, ds.
\]

We see that both functions \( N_0 \) and \( D_0 \) possess the left-hand and right-hand cuts in the \( s \) plane. The \( N_0(s, r) \) function has the following properties: it behaves as \( \exp 2ikr \) for \( r \to \infty \) and \( \text{Im} \, N_0(s, 0) = 0 \) for \( s > 0 \). With this last property for \( r = 0 \) after suitable deforming the contour \( C \) Eq. (1) reduces to the ordinary \( N_0/D_0 \) equation

\[
N_0(s, 0) = A_0(s) D_0(s, 0)
\]

with

\[
D_0(s, 0) = 1 - \frac{1}{\pi} \int_0^\infty \frac{\sqrt{(s') \, N_0(s', 0)}}{s' - s} \, ds'.
\]

The transition from Eq. (1) to the Marchenko equation is very easy. Bond states are preliminarily excluded. Let us rotate the integration path so that it encloses the left-