Let \((\xi_n)\) be a sequence of independent, identically distributed random variables, 
\[E\xi_n = 0, \quad D\xi_n = 1\]. We set \(S_0 = 0, \quad S_n = \frac{k}{\sqrt{n}} \xi_k\), and let \(P_n\) be the distribution in 
\(C[0,1]\) of the corresponding random polygonal line \(X_n(t), t \in [0,1]\) with vertices at 
the points \((\frac{k}{\sqrt{n}}, \frac{S_k}{\sqrt{n}})\), while \(P\) is the distribution of a standard process of 
Brownian motion \(W(t), t \in [0,1]\). In the paper one proves that under certain additional 
conditions imposed on the distribution of the variables \(\xi_n\) for any 
functional \(f\) from a very large class, the distributions \(P_n f^{-1}\) of the random 
variables \(f(X_n(.))\) converge in variation to the distribution \(P f^{-1}\) of the vari-
able \(f(W(.))\).

The well-known Donsker–Prokhorov principle (see, for example [1]) asserts that for 
\(n \to \infty\) the measures \(P_n\) converge weakly to the measure \(P\). The latter means that for a 
very wide class of functionals, namely for all \(P\)-almost everywhere continuous functionals 
\(f\) (i.e., measurable mappings of \(C[0,1]\) into \(R^t\)), the distributions \(P_n f^{-1}\) of the variables 
\(f(X_n(.))\) converge to the distribution \(P f^{-1}\) of the variable \(f(W(.))\).
There arises the natural question: can one obtain a similar statement regarding the behavior of the densities of the distributions \( P_n f_{-1} \) for a sufficiently large class of functionals? A more natural type of convergence of the densities is the convergence in the metric of \( L^4 \). Since this convergence is equivalent to the convergence in variation of the distribution themselves, our question can be formulated more precisely in the following manner: under what assumptions regarding the initial variables \( \xi_n \) can one assert that for a sufficiently large class of functional \( f \) the distributions \( P_n f_{-1} \) converge in variation to \( P f_{-1} \)?

Certainly, if one could establish that the measures \( P_n \) themselves converge in variation to \( P \), then the answer would be very simple: \( P_n f_{-1} \rightarrow \text{var} P f_{-1} \) for any measurable functional. However, as it can be easily seen, the measures \( P_n \) are always singular relative to \( P \) and this is exactly the fundamental difficulty.

In [3] the author has presented a method with the aid of which one has succeeded to make some progress in the solution of the formulated problem and to obtain the strong convergence of the distributions for functionals of the supremum type and for integral functionals. This method consists in the investigation of the limit behavior of the conditional distributions of the measures \( P_n \) on lines parallel to that direction with respect to which the variation of the functional \( f \) is sufficiently fast. In the present paper we make use of a similar technique in order to obtain the strong convergence of \( P_n f_{-1} \) to \( P f_{-1} \) for a significantly larger class of functionals \( f \).

We require a series of notations and concepts.

Let \( f \in C[0,1] \). We denote by \( \lambda(x) = (\lambda_x) \), \( \lambda_x = \{ x + c \in R^i \} \), a decomposition of the space \( C[0,1] \) into lines which are parallel to \( l \). We assume that the function \( \lambda \) is different from zero. Let \( t_0 \in [0,1] \) be such that \( \lambda(t_0) \neq 0 \). The quotient space \( C[0,1] / \gamma \), denoted by \( T \), will be identified with the subspace of those \( x \) for which \( x(t_0) = 0 \).

Let \( f \) be a measurable mapping of \( C[0,1] \) into \( R^i \). We denote by \( f_x, x \in T \), the function defined on \( R^i \) by the equality \( f_x(c) = f(x + c) \). For an open set \( V \subset C[0,1] \) we set \( A_x = \{ c; x + c \in V \} \). If \( V \) is convex, then \( A_x \) is an interval.

In the case when the direction \( l \) depends on a parameter \( n \), \( l = l_n \), then the above introduced notations will be supplemented by an index \( n \) : \( f = f_{l_n}, f_x = f_{x,l_n}, A_x = A_{x,l_n} \).

For a function \( h : R^i \rightarrow R^i \), differentiable on a set \( A \), we set \( \| h \|_A = \sup_{A} |h(c)| + \sup_{A} |h'(c)| \).

We denote by \( \mu \) the factor measure for the measure \( P \) relative to the decomposition \( f \).

We shall say that the functional \( f \) belongs to the class \( B(\alpha,\beta,\xi) \) (here \( \xi \in C[0,1] \), \( \xi_n \rightarrow l \)) if there exists a convex neighborhood \( V_\delta \) of the point \( P(\partial V_\delta) = 0 \) such that

1. \( f_\infty, f_{x,n} \) are continuously differentiable on \( A_x \) (respectively on \( A_{x,l_n} \)) for \( \mu \)-almost all \( x \);
2. \( \dot{f}_x(c) \not= 0 \) for \( \mu \times m \)-almost all pairs \( (x,c) \) such that \( c \in A_x \) (here \( m \) is Lebesgue measure);
3. for \( \mu \)-almost all \( x \in T \) and for any sequence \( (x_k), x_k \in T, x_k \rightarrow x \) for \( k, n \rightarrow \infty \) we have

510