GEOMETRIC INTERPRETATION OF CHRONOMETRIC INVARIANTS
IN GENERAL THEORY OF RELATIVITY

The geometric interpretation of Zelmanov's chronometric invariants is given and their connection to observable quantities as well as the connection of Zelmanov's approach to the generally covariant Dehnen's method is found.

Let us introduce a coordinate system $x^i$ on Riemannian manifold $V_4$. At each point let be defined the basis of vectors $D^i = \delta^i_4$, vector $D_4$ being time-like with $D_4 D_4 = -1$. Let the remaining three vectors $D^a$ be projected onto the hypersurface that is orthogonal to $D_4$. Thus a new basis of vectors $B^a$ has been introduced. In the original coordinate system $x^i$ their components are

$$
B^i_0 = \delta^i_4 - \frac{g_{i4}}{g_{44}} \delta^4_k \delta^k_4 \sqrt{-g_{44}},
$$

$$
B^i_4 = \delta^i_4 - \frac{g_{i4}}{\sqrt{-g_{44}}} \delta^4_k \delta^k_4, \quad B^i_0 B^j_4 = \delta^i_4.
$$

The contravariant components of some tensor $T$ calculated in the basis $B^a$ (we mark them by a bar) are

$$
T^{..i} = T^{..i} B^i_0 = T^{..i},
$$

$$
T^{.4} = T^{.4} B^4_0 = -T^{.4} \frac{1}{\sqrt{-g_{44}}}.\n$$

We can see that contravariant components of $T$ in the basis $B^a$ are (apart from the sign in the second relation) identical with their chronometrically invariant components defined by Zelmanov [1] and later used by Novikov [2].

Zelmanov's 3-vectors $R^a$ are not directly observable [3]. We have to project them on arbitrary direction orthogonal to $B_4$, i.e. observable quantities are

$$
\bar{\eta}_{ab} R^a X^b; \quad \frac{1}{\sqrt{-g_{44}}} \bar{\eta}_{ab} g^{ab} = \delta^4_4,
$$

where $X$ is a unit vector orthogonal to $B_4$. Now, let us introduce the basis of vectors $X^a$ such that $X^4 = B^4$ and $X^a = B^a / |B^a|$.

For a vector $R$ the observable quantities are $-R^4$ and $\bar{\eta}_{ab} R^a X^b$; for them we obtain

$$
-R^4 = R \cdot (B^4) \quad \bar{\eta}_{ab} R^a X^b = (X_4).\n$$

1) Latin letters $i, k, ...$ range and sum over 1, 2, 3, 4, the Greek ones $a, b, ...$ range and sum over 1, 2, 3. The signature of fundamental form is (1, 1, 1, -1).
Letters to the Editor

We can see that the observable quantities are projections of the vector $R$ on the basis $X(i)$. They form the so called physical basis [4]. It is, however, important to say that the vectors $X(i)$ are not uniquely defined by the motion of an observer. Instead of them he may also use vectors

$$X^i_{(e)} = e^i_k X^k_{(e)} , \quad e^i_k = \begin{pmatrix} e^i_0 & 0 \\ 0 & 1 \end{pmatrix} ,$$

where $e^i_k$ is a matrix of a regular transformation preserving the unit magnitudes of basis vectors. An experimental physicist would probably prefer such a physical basis $X^i_{(e)}$ which would be (at a given point) orthogonal as well.

In analogy with the above consideration we can say that any symmetric tensor of the second order gives the following measurable quantities

$$T^{44} = T^{ik}X^i_{(4)}X^k_{(4)} = \varepsilon ,$$
$$- T^{44}X^i_{(p)} = T^{ik}X^i_{(p)}X^k_{(4)} = - \pi_{(p)} ,$$
$$T^{\alpha\beta}X^i_{(e)}X^j_{(e)} = T^{ik}X^i_{(e)}X^j_{(e)} = \tau_{(e)} .$$

If $T^{ik}$ is the material and symmetric energy-momentum tensor, then the quantities $\varepsilon$, $\pi_{(p)}$, and $\tau_{(e)}$ can be interpreted as observable and chronometrically invariant density of energy, observable and chronometrically invariant momentum density and observable and chronometrically invariant momentum current density.

In agreement with our foregoing considerations it is clear that the quantities $\varepsilon$, $\pi_{(p)}$, and $\tau_{(e)}$ are related to the elements $dV^0$ of a hypersurface which is orthogonal to the world line of an observer in his surroundings. We can therefore integrate them only if there exists a global hypersurface $V^0$ orthogonal to the observer's world lines. If we consider such hypersurface $V$ along which it is not possible to synchronize the clocks, it can happen that an observer $A$ in the element $dV^0_A$ will measure a part of the contribution which has already been measured by an observer $B$ in the element $dV^0_B$. For this reason, it is clear that the integrals $\int V^0 \sqrt{(t)} dV$ and $\int \pi_{(p)} \sqrt{(t)} dV$ have no meaning of conserved physical quantities, instead of them we must consider the integrals

$$E = \int_V T^{ik}X^i_{(4)} \sqrt{(t)} v_i dV ,$$
$$P^i = - \int_V T^{ik}X^i_{(e)} \sqrt{(t)} v_i dV ,$$

where $v_i$ is a unit vector normal to the space-like hypersurface $V$.  