ALGORITHM AND PROGRAM FOR DESIGN OF RECTANGULAR FOUNDATION SLABS UNDER A GROUP OF COLUMNS

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Our proposed slab design method has been realized as a program for the Minsk-22 computer; it yields approximate solutions for flexible foundation slabs carrying the load of a group of columns. In contrast to the approach used in [3], we assume that the loaded areas can be fairly small and relatively far apart.

Foundation behavior is represented by the settlement matrix, which is a table of the deflections δ_{ij} of the soil surface at points i resulting from application of unit force at point j. The algorithm provides for the calculation of the δ_{ij} for various models (elastic layer, half-space) depending on the initial data, and for specification of the prepared settlement matrix, obtained in any other way, as part of the initial data. This makes it possible to design slabs on an arbitrary linear elastic foundation [1].

Let the slab dimensions be 2a × 2b × h. As with the ARAP-1 program [2], we solve for the deflections w by the method of least squares for polynomials of the form

\[ w = \sum A_{mn} x^m y^n, \]  

which approximately satisfy the biharmonic equation for slab flexure,

\[ L(w) = 0 \]  

and the two boundary conditions at each of the sides,

\[ I(w) = 0. \]

Here \( L(w) = \nabla^2 w + (q-p)/D \), where \( q \) is the load on the slab; \( p \) is the foundation reaction; \( I \) are certain linear operators that ensure satisfaction of the necessary conditions at the slab edges. As was pointed out in [3], this method does not yield a general solution for all plates, since a polynomial of the form (1) that is of restricted degree (we take \( m + n \leq 10 \)) cannot describe the multiwave deformed surface of the slab in our problem.

We use the following approach to extend the method to the general case: in the slab region, we isolate the zones within which the deflection function and its derivatives decrease rapidly. For a thin plate these will obviously be the areas at which forces are applied.

The solution is obtained separately for these zones by the method of [2] and added to the solution for the rest of the slab. The total solution is represented as the sum of N solu-
Along "B" axis

Along "7" axis

Fig. 2. Slab on elastic layer. a) Slab plan and loading diagram (the numbers give the loads in tons); b) moment and deflection diagrams.

tions, where \( N \) is the number of loads acting on the slab. For a certain \( k \)-th loading the problem splits into two parts (Fig. 1):

1. \( L(w_k) = 0 \) for \( l(w_k) = 0 \) on the slab contour;
2. \( L(w_k^*) = 0 \) for \( l(w_k^*) = 0 \) on the boundary of the region.

Here \( w_k(x, y) \) is the elastic surface of the slab that is outside the loaded zones; \( w_k^*(x, y) \) is the elastic surface of the part of the slab that is in the zone \( S_k \) experiencing the load.

The physical interpretation of this formulation is clear from Fig. 1. The solution is represented by two surfaces \( w_k(x, y) \) and \( w_k^*(x, y) \), each of which is smooth. In the zone \( S_k \) the elastic surface \( w_k^* \) is obtained as a solution to the problem of a plate carrying a uniformly distributed load. Here the slab deflections and moments are smooth surfaces that can be described accurately by a polynomial.

We introduce a step function,

\[
\zeta(t) = \begin{cases} 
0 & \text{for } t < 0, \\
1 & \text{for } t > 0 
\end{cases}
\]

and construct a new function with the self-evident properties

\[
\delta_i (x, y) = \delta [x - (x_i - a_k)] \delta [y - (y_i - b_k)] \\
\times \delta [y - (y_i - b_k)] \\
= \begin{cases} 
0 & \text{outside } S_k, \\
1 & \text{inside } S_k, 
\end{cases}
\]

Then

\[
w = [1 - \delta_i (x, y)] w_k + \delta_i (x, y) w_k^* = \\
= [1 - \delta_i (x, y)] \left[ \sum_{i,j=0}^{m,n} A_{ij} x^i y^j \right]_{S_k} + \delta_i (x, y) \left[ \sum_{i,j=0}^{m,n} A'_{ij} x^i y^j \right]_{S_k}
\]