ANALYSIS OF PILES SUBJECTED TO BUCKLING AND LATERAL BENDING

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Snitko and Snitko [1, 2] have studied the analysis of flexible supports (including piles) under the combined action of vertical and horizontal forces, as well as of moments applied on these supports at the ground surface elevation; in [1] it is assumed that the modulus of subgrade reaction varies with depth as a triangle, whereas in [2] it is assumed that this modulus varies as a parabola, the tangent to which at a point at the ground surface is vertical. For both types of variation of the modulus of subgrade reaction, differential equations of the elastic curve of the flexible support are set up, and their solutions are found in the form of equations by the method of initial parameters.

A wider formulation of this problem is presented in [3], in which the pile is divided into portions, within the limits of which the modulus of subgrade reaction is assumed to be constant. Using the condition of compatibility of the deformations at the boundaries of the individual portions, the author finds the influence functions of the initial parameters.

In this article, the writer presents a new method of analysis of piles subjected to simultaneous buckling and lateral bending, which is applicable under any law of variation of the modulus of subgrade reaction along the pile, of the longitudinal force in the transverse sections of the pile, and of the flexural rigidity. Unlike the procedures described in [1-3], this method permits analyzing piles whose upper ends project above the ground surface. It is a compact method which, thanks to a matrix formulation, can be easily programmed for computer solution.

Let us consider a pile driven into the soil to a depth h and loaded at the top (initial) section, which is located at distance l from the ground surface, by a vertical (axial) force P, a horizontal force Q, and a moment M (Fig. 1). Use will be made of sign rules which become evident if account is taken of the fact that for the initial pile section shown in Fig. 1, the horizontal deflection y, the bending moment M, and the horizontal force Q are positive, whereas the angle of rotation y is negative. On the basis of this sign rule, using the well-known solution of the problem of simultaneous buckling and lateral bending of a bar, we write the relation between the factors (y, y, M, and Q) in the initial section and the corresponding factors (y, y, M, and Q) in the section at the ground surface:

\[ N_t = F_t N_0, \]

*Here and in what follows it will be assumed that the shear force Q and the longitudinal force N are oriented with respect to the axis of the undeformed pile. With given values of Q and N, it is easy to find the shear \( \bar{Q} \) and longitudinal \( \bar{N} \) forces oriented with respect to the axis of the deformed pile:

\[ \bar{Q} = Q \cos \phi - N \sin \phi; \]

\[ \bar{N} = Q \sin \phi + N \cos \phi, \]

in which \( \phi \) is the angle of rotation of the pile section at which the internal forces are determined.


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in which

\[
F_0 = \begin{bmatrix}
1 & \sin a l_0 & 1 - \cos a l_0 & \alpha l_0 - \sin a l_0 \\
\alpha & \alpha^2 EI & 1 - \cos a l_0 & 0 \\
0 & \cos a l_0 & 0 & 0 \\
0 & -\alpha EI \sin a l_0 & \cos a l_0 & \frac{a l_0}{1}
\end{bmatrix}
\]

where \( EI \) is the flexural rigidity of the pile in the portion located above the ground surface; and

\[
a = \sqrt{\frac{P}{EI}}.
\]

Let us divide the part of the pile located below the ground surface into a sufficiently large number \( n \) of portions (Fig. 2), each of them with length \( \Delta z = h/n \). Let us also establish the relation between the factors in any two adjacent sections, for example in the \((i-1)\)th section \((y_{i-1}, \varphi_{i-1}, M_{i-1}, \text{and } Q_{i-1})\) and in the \(i\)th section \((y_i, \varphi_i, M_i, \text{and } Q_i)\). Since the number \( n \) is sufficiently large, the required relation can be expressed approximately as

\[
\begin{align*}
y_i &= y_{i-1} + \gamma_{i-1} \Delta z; \\
\varphi_i &= \varphi_{i-1} + \varphi_{i-1} \Delta z; \\
M_i &= M_{i-1} + M_{i-1} \Delta z; \\
Q_i &= Q_{i-1} + Q_{i-1} \Delta z.
\end{align*}
\]

(2)

Using well-known differential relations for bars subjected to buckling and lateral bending, we obtain

\[
\begin{align*}
y'_{i-1} &= \varphi_{i-1}; \\
\varphi'_{i-1} &= \varphi_{i-1} = \frac{M_{i-1}}{(EI)_{i-1}}; \\
M'_{i-1} &= Q_{i-1} - N_{i-1} \varphi_{i-1}; \\
Q'_{i-1} &= q_{i-1} = -Kx_{i-1} \gamma_{i-1}.
\end{align*}
\]

(3)

in which \((EI)_{i-1}\) is the flexural rigidity of the \((i-1)\)th pile section; \(N_{i-1}\) is the longitudinal force in the \((i-1)\)th pile section, equal to the difference between the force \( P \) and the resultant of the frictional forces acting on the lateral surface of the pile in the portion between the ground surface and the \((i-1)\)th section; \(z_{i-1}\) is the depth at which the \((i-1)\)th section is located, measured from the ground surface; \(q_{i-1}\) is the magnitude of the soil reaction on the pile at a depth \(z_{i-1}\); and \(K_{x_{i-1}}\) is the product of the modulus of subgrade reaction, at depth \(z_{i-1}\), by the pile width.