This marks the 80th year since publication of a special lecture by the author of one of the first domestic courses on foundations and foundation beds: Valerian Ivanovich Kurdyumov [1]. Kurdyumov was a professor at the Institute of Communication Engineers (now the V. N. Obraztsov Leningrad Institute of Railroad Transportation Engineers (LIIZhT)), and in his lecture "The Resistance of Natural Foundation Beds" he was the first to propose the use of photography in determining the boundary of the zone and the nature of displacement of ground beneath the weight of foundation models [2]. He wrote: "If we photograph over a period of several seconds sand in a vessel with a glass wall, subjected to crushing by a rod ..., the sand grains that remain at rest will appear sharp and clear in the picture, whereas the image of grains that moved during the exposure should not be sharp, but should appear smeared. The entire set of smeared images must define the shape of the prism of protrusion, and the boundary between smeared and sharp images must be the slip curve of the prism of protrusion" [2].

The method proposed by Kurdyumov, called the photorecording method, has proved to be very effective and has been used since by many investigators in this country and abroad. The method has been used also for recording the outline of the elastic core beneath a test plate, for which the camera has been rigidly fastened to the plate and has been displaced with it during the course of the experiment [3]. In this test, the sand grains in the core appear clear on the negative, but the grains in the remaining zone appear smeared. By slip lines Kurdyumov meant the "geometrical locus of slip planes of stress ellipses at different points in an unconsolidated body" [2]. The theory of limit in an unconsolidated medium equilibrium, which has developed in our time, especially through the work of V. V. Sokolovskii, V. G. Berezantsev, and others, also uses the term "slip lines," by which is meant lines tangent to areas of slippage [4]. Along these lines the condition of the Coulomb limiting state is fulfilled:

$$\tau_n = \sigma_n \tan \phi + k,$$

where $\tau_n$ and $\sigma_n$ are the tangential and normal stresses, respectively, acting on the investigated area of shear with the normal $n$; $\phi$ is the angle of internal friction; and $k$ is specific cohesion. The slip lines form two isogonal families. In the theory of limit equilibrium, the concept of slip lines is purely static, although it is perfectly logical that relative displacement should be observed along areas where the tangential stress attains a value at which condition (1) is fulfilled, since in other directions

$$\tau_n < \sigma_n \tan \phi + k.$$

Since there are two families of slip lines, Geniev [5] believes that visible displacements take place along one of them.
Fig. 2. Protrusion scheme according to Prandtl-Shield, \( \mu = \pi/4 - \rho/2 \).

Fig. 3. Directions of coordinate axes. a) Directions of coordinate axes and displacement path \( S \); b) principal axis of maximum stress \( \sigma_1 \) and surfaces of slippage; 1 and 2) first and second families of slip lines respectively.

which he calls active. This leads to the conclusion that "active" areas of slippage should coincide with the direction of maximum rates of shear strain. In order to relate stresses to strain rate, the hypothesis of plasticity potential has been proposed [6, 7], according to which the principal stress axes and axes of strain rates are considered to be coaxial. Shield [2] adopted as the plasticity potential a function coinciding with Mohr's equation of strength and solved the problem of squeezing of ground from under a foundation. According to his solution, the bearing capacity of a foundation bed remains the same as for Prandtl [4], and the rate vectors in the zone of maximum stress state do not coincide with slip lines but deviate from it by an angle \( \rho \) (Fig. 2). This latter causes us to doubt the correctness of Shield's solution [9], discussed later by Solov'ev [10].

In connection with this analysis, there is a question concerning what is obtained on a photograph when we use the photorecording method or similar methods by means of smoked glass, paraffin screen, and others.

Below we shall start from the position that the method of photorecording permits us to see not the lines of maximum strain rate but the paths of displacement of soil particles.

Let us designate by \( u_x \) and \( u_y \) the projections on the \( x \) and \( y \) axes of the total displacement of point \( M_1 \) (Fig. 3a) in the zone \( \Omega \) for time \( t \), and by \( v_x \) and \( v_y \) the corresponding velocities (rates). Since we are examining a process of two-dimensional plastic flow, the flow is assumed to be constant or proportional to a time function, such as the displacement rate of part of the boundary of zone \( \Omega \). For time \( t \) point \( M_1 \) is displaced to \( M'_1 \), \( M_2 \) to \( M'_2 \), and so forth. On the photograph we discover a curve \( S \), and the vectors of total displacement will be directed along the tangents to it. Consequently, for curve \( S \)

\[
\tan \alpha = \frac{dy}{dx} = \frac{u_y}{u_x} = \frac{v_y}{v_x},
\]

whence

\[
y = \int \frac{v_y}{v_x} \, dx.
\]

The equation of slip lines that we obtain from the condition of coaxiality of the stress and strain fields, being fulfilled in the presence of plasticity potential [7], is derived in the following way. Let us specify

\[
\frac{\sigma_x - \sigma_y}{2 \tau_{xy}} = \frac{\dot{\varepsilon}_x - \dot{\varepsilon}_y}{2 \gamma_{xy}} = \frac{\partial \sigma_x}{\partial x} - \frac{\partial \sigma_y}{\partial y} = \frac{\partial \gamma_{xy}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial x} = \gamma_{1},
\]

where \( \sigma_x, \sigma_y, \) and \( \tau_{xy} \) are the components of stress, and \( \dot{\varepsilon}_x, \dot{\varepsilon}_y, \) and \( \gamma_{xy} \) are components of strain rate. Following [4] we have