OPTIMAL CAPITAL ACCUMULATION IN GENERALIZED LEONTIEF MODELS**

BY

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"In the theory of allocation policy it need not be assumed that efficient allocation is the unique, unreservedly desired goal. Its being a policy objective does not preclude that policy makers and members of the public will want to weigh it against competing aims."

P. Hennipman in Relevance and Precision (Essays in Honour of Pieter de Wolff), 1976.

1 INTRODUCTION

The dynamic Leontief model, characterised by constant input-output and constant capital coefficients, belongs to the standard tools of economic analysis. Its simplicity, as compared with, for example, the Von Neumann model makes it more suitable for empirical application. A recent example is given by Tsukui and Murakami [9]. Building upon earlier work, the authors present paths of optimal capital accumulation for the Japanese economy.

For the time being, our aim is more modest. In this paper we shall analyse paths of optimal accumulation in this type of model by means of numerical examples. This approach has the advantage that the dynamic characteristics can be shown more clearly. More specifically, we shall pay attention to the influence of a time preference factor, the meaning of stock activities and the heterogeneity of capital with respect to the available techniques. It goes without saying that numerical linear programming examples are no more than illustrations. However, we think that such a middle-of-the-road approach may have its own merits along with abstract theorizing on the one hand and practical applications on the other hand.

In programming models a choice has to be made with regard to the objective function and the relevant constraints on economic growth. In all our examples, discounted per capita consumption will be maximized over a finite

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planning horizon. The Leontief model is generalized in the sense that there is a choice with regard to the linear technology. Labour will be the only non-produced factor of production. In case of a finite horizon terminal conditions must be specified to obtain a solution. Fortunately, this requirement is not very restrictive. Due to the so-called turnpike property of Leontief models, the optimal solutions do not depend heavily on terminal conditions except for some periods near the end of the period.

In section 2 a general outline of the model will be presented. The turnpike property of the system will be illustrated for different discount rates in section 3. In section 4, so-called stock activities will be introduced to prepare the ground for a model with heterogeneous capital per category of goods. The heterogeneity of capital relates to different techniques. More specifically, it is assumed that production according to a certain technique is only possible with the appropriate type of capital goods. This extension of the traditional Leontief model is analysed in section 5. In all cases the same numerical values for the parameters will be chosen, so that the results can be compared with each other. In the final section of the paper a few remarks will be made with regard to the relevance of this study.

Our treatment of the problem of optimal accumulation in Leontief models differs from the approach by Tsukui and Murakami [9] in a number of ways. First, more attention is paid to shadow prices, especially in sections 2 and 3. Second, it is shown in section 4 that the introduction of stock activities may lead to unrealistic results. Third, the model with embodied technical change presented in section 5 may perhaps be considered as something new.

2 A MODEL OF OPTIMAL CAPITAL ACCUMULATION

Let A and B be square matrices of fixed input-output and capital coefficients representing the given technology, respectively. Writing \( x(t) \) for the vector of gross production and \( f(t) \) for the vector of consumption levels the following relation holds:

\[
(I-A)x(t) - B[x(t + 1) - x(t)] - f(t) \geq 0
\]

The equation states that the surplus of production (first term) can be used for

1 A list of main symbols is presented in the appendix.