THE NONLINEAR THEORY OF PLASMA INSTABILITY IN AN OSCILLATING DISCHARGE

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It was demonstrated in paper [1] that a system consisting of a cold spatially homogeneous plasma and hot electrons, inserted in a homogeneous magnetic field, is unstable against potential oscillations with a characteristic frequency of \( \omega \ll \Omega \) \( \Omega \) — the cyclotron frequency of ions) on the assumption that the density of hot electrons is inhomogeneous. For \( \alpha = n_{01}/n_0 \ll 1 \), where \( n_0, n_{01} \) are the equilibrium densities of cold and hot plasma, respectively, the maximum increment is much smaller than the frequency. Such a system is realized, for example, by an oscillating discharge [2].

In what follows, we shall present an analysis of the nonlinear effects resulting from such instability. In view of the fact that the increment of the oscillations is much smaller than their characteristic frequency, the nonlinear effects may be described by equations of the theory of weak turbulence [3, 4].

Let us first formulate the assumption made in the discussion of our problem. In a homogeneous magnetic field \( H = H_z \) there exists a mixture of an isothermal spatially homogeneous plasma with Maxwellian distribution in the velocities \( (T_e = T_i = \text{const.}, \ n_0 = \text{const.}) \) and “hot” electrons (also with Maxwellian distribution in the velocities, \( T_{e1} = \text{const.}, \ T_i \ll T_{e1} \)), the equilibrium density \( n_{01} \) of which varies along the \( y \)-axis. We shall consider potential oscillations of the type \( \exp \{-i\omega t + ik \cdot r\} \) (assuming the quasi-classical approximation to be applicable) with characteristic frequencies \( \omega \ll \Omega \).

We may obtain the initial equations for a description of the nonlinear effects in such a system through the use of a method evolved in [3, 4]. To accomplish this, we must solve Vlasov’s kinetic equation for the oscillating part of the distribution function of electrons and ions of the “cold” plasma with an accuracy to terms of the third order, inclusive, with respect to the amplitude of the potential. For the oscillating part of the distribution function of hot electrons it is sufficient merely to take a solution of Vlasov’s linearized equation because, as can be shown by additional analysis, contributions from hot electrons are negligible in the nonlinear terms compared with those of the cold plasma.

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We can find — using the method of successive approximations — that the oscillating part of the distribution functions of ions and electrons of the cold plasma has the form [5]

\[ f^{(1)}_{k\omega} + f^{(2)}_{k\omega} + f^{(3)}_{k\omega}, \]

where

\[ f^{(1)}_{k\omega} = -\frac{\epsilon_0}{T} \varphi_{k\omega} \left( 1 - \omega \right) J_0(\xi_k) \exp \left\{ i\xi_k \sin \left( \phi - \Psi_k \right) \right\}, \]

\[ f^{(2)}_{k\omega} = \sum_{k_,\omega'} M_{k\omega,k_\omega'} \left( \phi_{k\omega'} \phi_{k'\omega'-\omega} - \langle \phi_{k\omega'} \phi_{k'\omega'-\omega} \rangle \right), \]

\[ f^{(3)}_{k\omega} = \sum_{k_,\omega'} L_{k\omega,k_\omega'} \left( \phi_{k\omega'} \phi_{k'\omega'-\omega} - \langle \phi_{k\omega'} \phi_{k'\omega'-\omega} \rangle \right), \]

\[ M_{k\omega,k_\omega'} = -\frac{i\epsilon_0}{T} J_0(\xi_k) J_0(\xi_{k'-k}) \exp \left\{ i\xi_k \sin \left( \phi - \Psi_k \right) \right\} \left\{ \langle k, k' \rangle \left( \omega - \omega' - (k_z - k_{z'}) v_z \right) \right\}, \]

\[ L_{k\omega,k_\omega'} = -\frac{i\epsilon_0}{T} J_0(\xi_k) J_0(\xi_{k'-k}) \exp \left\{ i\xi_k \sin \left( \phi - \Psi_k \right) \right\} \left\{ \langle k, k' \rangle \left( \omega - \omega' - (k_z - k_{z'}) v_z \right) \right\}, \]

where we have introduced cylindrical coordinates in both the velocity space \((v_x, \phi, v_z)\) and the space of wave vectors \((k_x, k_y, k_z)\).

\[ \xi_k = k_x v_x, \quad f_0 is the Maxwellian distribution function in the velocities, \varphi_{k\omega} the Fourier component of the potential, \quad J_0 the Bessel function, \quad \langle \ldots \rangle denotes averaging over the set of oscillations (chaotic phase). \]

The resonance denominators are written so that, in order to effect correct bypassing of the poles, they must be supplemented by the factor \(i\delta \left( \Delta > 0, \Delta \rightarrow 0 \right)\).

The oscillating part of the distribution function of hot electrons is given by the expression [1]

\[ f^{(1)}_{h\omega} = -\frac{\epsilon_0}{T} f_{01} \left( 1 - \omega - \omega^* \right), \]

where \(f_{01}\) is the Maxwellian distribution function in the velocities, \(\omega^* = -k_x T_{e1} \kappa/m_e \Omega, \kappa = \delta \log_{10} \left( \frac{\delta}{\delta y} \right)\).

By introducing expression (1) to (7) into Poisson’s equation and using the standard method [3, 4], we obtain the equation of weak turbulence in the form