I. Introduction:
The aggregation of individual preferences into a social preference ordering is one of the most fundamental of the problems in social science. The general problem is non-trivial even for a small group of individuals. Given plausible conditions which a social preference ordering should satisfy, Arrow demonstrates in his classic work [1] that even when individual preferences satisfy reasonable conditions there need not exist a general social welfare function or preference ordering if the number of involved persons and alternatives under consideration is greater than or equal to three. Only in the special case of two alternatives is the device of decision by majority rule proven to provide a generally satisfactory social preference ordering for an arbitrary number of individuals. 2

II. The Basis for the Model:
This paper presents a mathematical model of elections which allows certain aspects of the problem of social choice to be systematically explored. Davis and Hinich [5, 6] report this basic model in detail and derive more limited and restrictive versions of a part of the results reported below. The model may be viewed in either a normative or a positive light. Basically, it consists of two parts. First, there is a simple rule which is taken to represent the decision mechanism of a voter confronted with choosing between competing candidates who are viewed as being nothing more than a set of policies. Second, there is a probabilistic representation of the distribution of preferences across the population. For most of the results presented in this paper the distribution of preferences is assumed to be continuous. This assumption is convenient and it would appear to be a suitable approximation for large populations. It does imply that the voting population is at least infinite. It should be pointed out that issues, though not necessarily utility, are measured on a cardinal scale which is the usual practice in economic
models where goods are given cardinal measures. However, utility (or disutility) need not have a cardinal measure although for certain limited purposes it is convenient and insightful to assume such a measure.

It is appropriate to begin the discussion of the formal model by considering individual preferences. Assume that each individual in the voting population perceives n issues which are relevant to any public choice such as an election. Each voter is assumed to have a most desired or most preferred position on each of these issues. Thus the column vector \( x' = (x_1, \ldots, x_n) \) denotes the most preferred position of a given voter for each of the relevant issues. In other words, if the \( i^{th} \) issue is monetary support of the government for higher education, then \( x_1 \) represents the number of dollars which the given individual desires to see the government spend on higher education. Since any issue, such as governmental support for higher education, can be divided into sub issues and since different individuals can perceive the sub issues in various ways, the assumption that \( n \) dimensional vectors, whose components are measured on a cardinal scale, can be taken to represent the most preferred position of individual voters is rather restrictive. However, if one constrains oneself to analyzing major issues and if the cardinal scaling is not pushed too far, then it can be argued that this assumption is not prohibitively restrictive. It is important to note that each individual voter in the population is allowed to have a different most preferred position and that infinities of such positions are available.

Obviously, except in the most trivial case, all voters cannot be allowed to enjoy their most preferred position. If a voter is required to select between only two competing positions, then it is assumed that he will choose and prefer a position \( \psi' = (\psi_1, \ldots, \psi_n) \) over a position \( \psi' = (\psi_1, \ldots, \psi_n) \) if and only if

\[
\|x-\psi\|^2_A < \|x-\psi\|^2_A
\]  

(1)

where

\[
\|x-\psi\|^2_A = (x-\psi)'A(x-\psi)
\]

\[
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\]

and \( A \) is a positive definite \( n \times n \) matrix. The quantity \( \|x-\psi\|^2_A \) can be considered the utility loss that a given voter feels if the position \( \psi \) is enacted into policy instead of his preferred position \( x \). This loss function is a quadratic in \( \psi \). The matrix \( A \) is called the loss