Multilevel optimization of laminated composite structures

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Abstract  A two-stage optimization method aiming at the optimal design of shells and plates made of laminated composites has been developed. It is based on a mixture of sensitivity analysis, optimality criteria and mathematical programming techniques. The design variables are the optimality criteria and mathematical programming techniques. The design variables are the macro-element thicknesses and the layers' angles. Weight minimization with material efficiency maximization are the objectives with constraints on stresses and displacements. Maximization of the material efficiency is performed at one level using the conjugated method applied to the angles of the macro-element layers keeping the thicknesses constant. The other level is dedicated to weight reduction using optimality criteria and using as variables the macro-element thicknesses with the angles of the macro-element layers constant.

1 Problem definition
Methods usually employed in structural optimization have frequently faced implementation difficulties and efficiency gaps when applied to composite materials. Some possible causes are the number of design variables and constraints that may become very large. Several researchers have intended to solve the problem while developing multilevel optimization algorithms based on substructuring techniques (e.g. Schmit and Mehrinfar 1981; Watkins 1986; Wei and Boohua 1987; Sadr et al. 1989).

The present work uses decomposition techniques to address the problem at two levels expecting a global efficiency improvement based on the size reduction for each stage. The types of structures studied were plates and shells, which are the most common in laminated composite materials.

The plate and shell structural analysis is carried out using the finite element displacement method. The element type adopted is the tridimensional degenerated shell element considering material anisotropy and structural layout of the layer. It is an isoparametric element with eight nodes and five degrees of freedom per node (Fig. 1) based on the Mindlin shell theory.

2 Problem formulation
Taking advantage of substructuring techniques, each structure is divided into macro-elements. The number and shape of the macro-elements are defined when elaborating the structural model and depend mainly on the degree accuracy of the analysis. All members of each macro-element have the same characteristics, such as thickness, number of layers and stacking sequence, and thickness continuity is imposed at the macro-element boundaries.

The design variables are the macro-element thickness, \( t_m \), and the fiber layer angles, \( \theta_{i,m} \), \( i = 1, \ldots, N_m \), where \( N_m \) is the number of layers of the macro-element. The stacking sequence is an implicit variable since there is independency between the fiber angles of the different layers.

The objective function is the weight of the structure, assuming that the total thickness is not significant compared with the other shell or plate dimensions,

\[
W(t) = \sum_{k=1}^{M} \Omega_k \rho_k t_k, \tag{1}
\]

where \( M \) is the number of macro-elements, \( \Omega_k \) the average macro-element superficial area and \( \rho_k \) the specific weight of the \( k \)-th macro-element. The adopted constraints are imposed limits on layer stresses, based on the failure criteria, and on global displacements. The stress constraints have the following form:

\[
g_j = 1 - \frac{R_j'}{R_0} \leq 0, \quad j = 1, \ldots, N_s, \tag{2}
\]

where \( R_{kj} \) is the strength parameter, calculated as the ratio between the failure stress and the actual stress, \( R_0 \) is a safety factor and \( N_s \) is the number of stress constraints.

The strength parameter \( R_{kj} \) is a function of the actual stresses and is obtained using the interactive quadratic failure criteria of Tsai-Wu (Tsai and Hahn 1980; Tsai 1987),

\[
[F_{ik} \sigma_i \sigma_k] R_j^2 + [F_{ik} \sigma_k] R_j = 1, \tag{3}
\]

where \( i \) and \( k \) take the values 1, 2 and 6, and \( R_j \) is the ratio between the maximum allowable stress and the actual stress.
correspondent to the ply $j$.

The displacement constraints are expressed by

$$g_j = \frac{u_r}{u_0} - 1 \leq 0, \quad j = 1, \ldots, N_d,$$

where $u_r$ is the global displacement $r$, and $u_0$ is the absolute maximum allowable displacement and $N_d$ is the number of displacement constraints.

The final formulation of the present structural optimization is

$$\min W(t),$$

subject to

$$g_j(\theta, t) < 0, \quad j = 1, \ldots, N_d + N_s,$$

where $\theta$ is the vector of layer angles and $t$ the vector of macroelement thicknesses.

### 3 Optimization algorithm

The strategy adopted was to decompose the optimization algorithm in two levels. These suboptimization problems were chosen as possibly independent stages aiming at dealing with two reduced subproblems and the optimal solution is searched for by iterating between the two levels.

The first level is dedicated to maximizing the material efficiency using only the layer angles as variables. The second level consists of minimizing the weight considering the layer thicknesses as variables. In effect, the weight function does not vary with the layer angles, which affect only the constraints.

The formulation for the first level is

$$\min F(\theta) = \min \tilde{g}(\theta, t_0) = \min \left[ \max (g_j, \quad j = 1, \ldots, N_d + N_s) \right],$$

where the minimization of $F(\theta)$ is an unconstrained problem and $t_0$ is the thickness vector kept constant during this suboptimization level. At this level, the changes of the layer angles create modifications of stress and displacement fields. If the initial solution is not feasible, at the end of this level, it will be closer to, or inside, the feasible region. On the other hand, if it is feasible, then at the end of this level it will be a better design.

The second level may be stated as

$$\min W(t),$$

subject to

$$g_j(\theta_0, t) \leq 0, \quad j = 1, \ldots, N_d + N_s,$$

where the variables are only the macro-element thicknesses and the layer angles are kept constant. At this level, it is intended to minimize the weight and bring the value of at least one of the constraints close to zero.

The optimization techniques are different for each level such that at the first level, the conjugate gradient method was adopted and at the second level an optimality criterion was chosen. For both levels, the sensitivity analysis was evaluated using the adjoint structure method. The conjugate gradient method of the first level used was that presented by Polak and Ribiere (1971). The algorithm is stopped if any of the global convergence conditions, $C_1$ or $C_2$, are verified or if the number of cycles exceeds a control parameter $I_3$. The optimization sequence may be graphically represented, as shown in Fig. 2, and the basic flowchart is presented in Fig. 3.

The optimality criteria were developed based on the optimality condition that the constraint with the greater value becomes active,

$$\bar{g} = 0.$$ 

Indeed it was considered that the absolute value of the constraint with the greater value is decreased during the iteration history. This is represented by