Optimal locations of internal line supports for rectangular plates against buckling

C.M. Wang
Department of Civil Engineering, National University of Singapore, Kent Ridge, Singapore

K.M. Liew
School of Mechanical and Production Engineering, Nanyang Technological University, Nanyang Avenue, Singapore

L. Wang and K.K. Ang
Department of Civil Engineering, National University of Singapore, Kent Ridge, Singapore

Abstract This paper concerns the optimal locations of straight line internal supports for rectangular thin plates, under uniaxial compression, so as to maximize the elastic buckling loads. The optimization problem can be readily solved by using the newly developed pb-2 Rayleigh-Ritz method for the buckling analysis. Since the method avoids any discretization, the problems associated with the need to move the nodal points so as to coincide with the internal line support are eliminated. Moreover, the optimization exercise may be performed on a personal computer since the method requires a relatively smaller memory and computational effort as a result of a lower number of unknown variables involved when compared to other discretization methods such as the finite difference or finite element methods. Optimal results will be presented for plates with various combinations of edge conditions, aspects ratios and one or two internal line supports. The results indicate large sensitivity of buckling loads to supporting conditions.

1 Introduction
The stiffness and load carrying capacity of structures spanning or covering large areas may be increased significantly by introducing internal supports. When designers are given the choice of placing the supports at any position in the structure, it is worthwhile considering the optimization of their locations so as to maximize the structural performance. Even if this choice is somewhat restricted, such optimal solutions are still useful to designers because they provide a basis for measuring the efficiency of any other design and show the direction to be taken towards a more economical design.

There has been a considerable amount of research done on the optimization of internal support positions for beams (Rozvany 1974-1975; Prager and Rozvany 1975; Mroz and Rozvany 1975), columns (Rozvany and Mroz 1977; Olhoff and Taylor 1978), cables and arches (Thevendran and Wang 1985; Goh et al. 1987; Wang 1987). Goh et al. (1991) have even provided a unified numerical approach, based on a control parameterization technique, for solving cross-section structural optimization problems under general constraints and variable internal support positions. However, when one shifts the focus from one-dimensional structures to two- or three-dimensional structures such as plates or shells, there appears to be a dearth of papers in the open literature on such support optimization studies. The aim of this paper is to initiate the exploration of the optimal layout of internal supports for plates so as to maximize the elastic buckling loads. For a start, straight line internal supports and rectangular plates under uniform compression are considered.

The buckling analysis may be performed by any of the well-known discretization methods such as the finite element, finite difference or collocation method. These methods, however, would be very cumbersome when optimizing the internal support position as the discretization points or finite element mesh must have the capability to evolve so as to accommodate the restraint of zero deflection at the supports. Moreover, a large computer resource is necessary to handle the large number of unknown variables. In this paper, the newly developed pb-2 Rayleigh-Ritz method (Liew and Wang 1992a, b, c; Wang et al. 1992) will be employed for the analyses. This method facilitates the automation of the Rayleigh-Ritz method by using a Ritz function that is general in satisfying all the geometric boundary conditions of arbitrarily shaped plates. The pb-2 Ritz function is defined as the product of a two-dimensional polynomial function (denoted by the designated code p-2) and a basic/boundary function (denoted by the code b). This key basic function consists of the product of (1) the equations of the boundary edges, each raised to the power of either 0, 1 or 2, corresponding to free, simply supported or clamped edges and (2) the equations of the internal supporting lines. A similar Ritz function was also proposed by Rozvany and Laxon (1965) for bending analysis of plates with clamped and internal simply supported edges. Being a continuum method, there is no need for discretization or mesh generation. Consequently, the aforementioned problem may be solved more accurately and efficiently.

2 Problem formulation and method of solution
Consider a flat, isotropic, thin, elastic rectangular plate of constant thickness, t, and side lengths a and b. It is subjected to a uniform compressive force, $N_x$ as shown in Fig. 1. The
plate may have any combination of supporting edges and is internally supported by a prescribed number of internal line supports which span between two plate edges. The problem is to determine the optimal locations of these internal line supports so as to maximize the elastic buckling load. The strain energy stored by the plate during buckling is given by (Timoshenko and Gere 1961)

\[ J = \frac{E h^3}{12(1-\nu^2)} \int \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 \, dA, \]

in which \( x, y \) are the Cartesian coordinates with the origin at the plate centre, \( D = E h^3 / (12(1-\nu^2)) \) = flexural rigidity of the plate; \( \nu \) Poisson's ratio; \( w(x, y) \) the deflection of the middle plane of the plate perpendicular to the \( xy \) plane; \( A \) the area of the plate; \( dA = dx \, dy \).

The potential energy, \( V \), of the inplane normal load, \( N_x \), is given by (Timoshenko and Gere 1961)

\[ V = \frac{1}{2} \int_A N_x \left( \frac{\partial w}{\partial x} \right)^2 \, dA. \]  

The total potential energy functional, \( F \), is given by

\[ F = U + V. \]  

The transverse displacement surface may be parameterized by

\[ w(x, y) = \sum_{i=1}^{m} c_i \phi_i(x, y), \]

where \( c_i \)'s are the unknown coefficients to be varied and the Ritz function, \( \phi_i \), is formed by taking the product of the boundary and internal support functions, \( \phi_1 \) (which serves...