An Order-Algebraic Definition of Knuthian Semantics*.

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Abstract. This paper presents a formulation, within the framework of initial algebra semantics, of Knuthian semantic systems (K-systems) which contain both synthesized and inherited attributes. The approach is based on viewing the semantics of a derivation tree of a context-free grammar as a set of values, called an attribute valuation, assigned to the attributes of all its nodes. Any tree's attribute valuation which is consistent with the semantic rules of the K-system may be chosen as the semantics of that derivation tree.

The set of attribute valuations of a given tree is organized as a complete partially ordered set such that the semantic rules define a continuous transformation on this set. The least fixpoint of this transformation is chosen as the semantics of a given derivation tree. The mapping from derivation trees to their least fixpoint semantics is a homomorphism between certain algebras. Thus, the semantics of a K-system is an application of the Initial Algebra Semantics Principle of Goguen and Thatcher. This formulation permits a precise definition of K-systems, and generalizes Knuth's original formulation by defining the meaning of recursive (circular) semantic specifications.

The algebraic formulation of K-systems is applied to proving the equivalence of K-systems having the same underlying grammar. Such proofs may require verifying that a K-system possesses certain properties and to

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*This paper is a revision of a paper presented at the 17th Annual IEEE Symposium on Foundations of Computer Science, Houston, Texas, October 1976. This research was supported in part by National Science Foundation Grant No. MCS 03633 A04 and by U.S. Department of Energy Contract No. EY-76-S-03-0034, PA 214.

†Editor's Note: This paper is one of several presented in the Session on Semantics at the 17th Annual IEEE Symposium on Foundations of Computer Science, 1976, and subsequently invited for submission to this journal to present different approaches to the subject.
this end, a principle of structural induction on many-sorted algebras is formulated, justified, and applied.

1. Introduction

The purpose of this paper is to present an algebraic treatment of Knuth's approach to the specification of the semantics of context-free languages [14, 15, 16]. Knuthian systems, hereinafter called K-systems, have been called declarative semantics [8, 9, 32, 33], K-grammars [1] and attribute grammars [3, 13, 20, 21]. The latter term seems to dominate in the more recent literature, but it emphasizes a syntactic point of view. We view K-systems as Knuth originally intended, namely as a method for assigning meanings to derivation trees on context-free grammars. This viewpoint is in full agreement with the algebraic approach to semantics.

K-systems have been used in many Computer Science investigations, such as programming language translation [20], program correctness [10], program optimization [24], question-answering systems [25], semantics of programming languages such as SIMULA 67 [32] and PL360 [8], and programming language design [1]. Also, various formal models for programming language syntax and translation related to Knuthian semantics have been proposed [6, 17, 23, 30].

Goguen and Thatcher [11] have given an initial algebra formulation of K-systems in which only synthesized attributes can appear. They leave open the problem of including inherited attributes in an algebraic formulation of K-systems. We propose a solution to this open problem in this paper. Our solution, a formulation of K-systems within the framework of initial algebra semantics, combines the intuitive appeal of K-systems with the theoretical power of algebraic methods. This approach permits a simple and precise definition of K-systems, and generalizes Knuth's original formulation by defining the meaning of recursive (circular) semantic specifications. It has been used by Chirica [4] in proofs of correctness of programming language translators. Although we argue that it is possible to convert any K-system definition (including a circular one) into an equivalent algebraic definition, we examine the more useful alternative of using both methods for writing semantic definitions. We are thus freed from the combinatorial aspects of K-systems such as tree traversal, topological sorting and especially the circularity problem, while taking full advantage of their intuitive appeal. Reference [12], which is a more recent version of [11], mentions two unpublished suggestions for solving the problem discussed in this paper. One, by Goguen and Zamfir, is different than our approach; the other, by Burstall, appears to be more related to our work.

In this paper we assume a knowledge of K-systems, many-sorted (heterogeneous) algebras [2, 11], and a version of Scott's order-theoretic approach to computation that uses complete posets, e.g. [22]. Section 2 briefly reviews the initial algebra approach to semantics and the fundamentals of chain-complete posets. K-systems are discussed in Section 3. Section 4 presents an algebraic formulation and generalization of K-systems together with discussion and an example. In Section 5 we discuss the practical implications of the algebraic definition of K-systems, where we also argue in favor of the combined use of