DCOC: an optimality criteria method for large systems
Part I: theory*

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Abstract A highly efficient new method for the sizing optimization of large structural systems is introduced in this paper. The proposed technique uses new rigorous optimality criteria derived on the basis of the general methodology of the analytical school of structural optimization. The results represent a breakthrough in structural optimization in so far as the capability of OC and dual methods is increased by several orders of magnitude. This is because the Lagrange multipliers associated with the stress constraints are evaluated explicitly at the element level, and therefore, the size of the dual-type problem is determined only by the number of active displacement constraints which is usually small. The new optimality criteria method, termed DCOC, will be discussed in two parts. Part I gives the derivation of the relevant optimality criteria, the validity and efficiency of which are verified by simple test examples. A detailed description of the computational algorithm for structures subject to multiple displacement and stress constraints as well as several loading conditions is presented in Part II.

1 Introduction

Before explaining the authors’ motivation for this new development, the historical background of optimality criteria methods is outlined briefly. The modern history of structural optimization has been marked by the development of two rather independent mainstreams of research, promoted by the so-called analytical and numerical schools. The numerical school of structural optimization, dealing with computer-based structural synthesis, was founded over three decades ago by Schmit (1960), who was the first to show the feasibility of coupling finite element analysis and non-linear mathematical programming to create automated optimal design capabilities for a rather broad class of structural systems. During the past three decades, the central task of research activities in this field has been aimed at improving the efficiency of the general strategy used. The necessity for this improvement is due to the facts that
(A) mathematical programming algorithms need frequent calculations of the objective and constraint functions and their gradients for which very expensive structural and sensitivity analyses are required, and
(B) the computer time requirement of mathematical programming methods depends largely on the number of design variables which is usually very high for complex engineering structures.

In view of the above, two aspects should be considered for enhancing the efficiency of a specific algorithm:
1. the number of complete structural reanalyses needed for reaching the optimum, and
2. the computer time requirement for the redesign procedure.

Because of the first aspect, the approximation technique introduced by Schmit and his associates (Schmit and Farshi 1974; Schmit and Miura 1976) has become the dominant methodology of structural optimization. In this general approach, the very costly information obtained by structural and sensitivity analyses is used to construct an explicit approximate problem in which the behavioural constraints are typically linearized in terms of either the design variables or their reciprocals, or a mixture of both (Starnes and Haftka 1979). After finding an optimum for the approximate problem, an exact analysis is carried out and a new approximation is constructed. This way the number of exact analyses is reduced considerably.

The second aspect mentioned above concerns the efficiency of the optimization method (i.e. the optimizer) used for solving the approximate problem. As the number of design variables increases, this aspect can become more critical than the first aspect and hence it represents a limiting factor for the capability of a structural optimization algorithm. Discretized optimality criteria methods (DOC) proposed by aerospace scientists (Berke 1970; Venkayya, Khot and Berke 1973), and later in a unified form—known as dual methods—by Fleury (1979), improved this aspect of structural optimization algorithms. In these methods, separable convex approximate problems are transformed into quasi-nonconstrained dual problems in which the variables are Lagrangian multipliers corresponding to the behavioural constraints and hence the only constraints for the dual problem are the nonnegativity requirement of those dual variables. The number of active constraints is often smaller, in some cases much smaller, than the number of design variables. Therefore, the dual problem usually involves fewer variables and only trivial constraints, which gives it a distinct advantage over the primal MP methods for problems where convex separable approximation can be used.

The approximation methodology was generalized in recent years (Vanderplaats and Salajegheh 1988, 1989; Zhou 1989; Zhou and Xia 1990; Canfield 1990) for a much broader class of structures. These authors have realized that for more complex structural systems, such as frames or plates
or some geometrical optimization problems, the behavioural
constraints are more directly influenced by some intermedi-
ate quantities rather than the design variables themselves.
By linearizing the behavioural constraints or some interme-
diate responses in terms of those intermediate variables, the
quality of approximation is highly improved and, therefore,
the number of reanalyses is reduced.

Another important general methodology is decomposition
(Sobieszczanski-Sobieski et al. 1983, 1985), in which a large
optimization problem is converted into a set of much smaller,
separate but coordinated subproblems. This methodology
becomes increasingly important in the rapidly expanding field
of multi-disciplinary optimization.

An excellent review of methods of the numerical school is

Concerning the optimization capability of various algo-
rithms, it appeared until recently that the dual method had
reached the absolute limit of efficiency. It will be shown in
this paper, however, that a further rather dramatic improve-
ment is possible. A new class of optimality criteria methods
for cross-sectional optimization problems will be presented
on the basis of the general methodology developed by the
so-called "analytical school" of structural optimization.

The origin of the analytical approach is usually traced
back to the theory of least-weight trusses, developed by
Michell (1904) around the turn of the century. This field has
been developed very intensively since the sixties by Prager,
his associates and others (e.g. Prager and Shield 1967; Prager
and Taylor 1968; Masur 1970; Prager and Rozvany 1977;
Mróz 1972; Olhoff 1976). A rather broad class of structural
optimization problems have been investigated by the ana-
litical school, although closed form solutions were limited to
simple, idealized problems. The general methodology used in
this field is the so-called continuum-type optimality criteria
method (COC), which is based on the Euler-Lagrange type
minimality conditions in infinite dimensional design spaces,
derived from variational principles. An extensive treatment
of the COC method is given by Rozvany (1989) in his recent
book.

The fundamental feature of the COC method is that the
analysis equations are considered explicitly in the optimiza-
tion problem as equality constraints. This is a natural feature
of analytical investigations since no analyser is available for
deriving solutions automatically. For the cross-sectional op-
timization of static systems, the COC formulation has the
following characteristics.

(1) Displacement constraints are formulated by means of the
virtual work principle.

(2) The flexibility formulation for the real and virtual load
systems is used and the basic unknowns involved are
then the real and virtual forces together with the cross-
sectional parameters.

(3) Kinematical admissibility of the real force system is tem-
porarily omitted in the formulation of the original prob-
lem, since the stationarity conditions (i.e. necessary con-
ditions of optimality) imply automatically kinematical ad-
nmissibility. This also follows from the stationary mutual
energy theorem (Shield and Prager 1970; Huang 1971)
that states that among all statically admissible solutions,
the solution representing stationary mutual energy is also
kinematically admissible.

More recently, an iterative algorithm coupling the COC
methods and FE analysis for problems with stress constraints
and a single displacement constraint has been developed by
the authors (Rozvany and Zhou 1991a, 1991b; Zhou and Roz-
vany 1991). It has been shown that the COC methods are
extremely efficient in handling stress constraints since the
latter are uncoupled in the optimality criteria and the corre-
spanding Lagrangians can, therefore, be obtained explicitly
at the element level. The effect of active stress constraints
is represented by applying modified kinematic conditions for
the virtual load system corresponding to the displacement
constraint. For example, for beams with constant depth and
variable width, the virtual curvature has a given constant
value at the cross-section where the flexural stress constraint
is active (Rozvany and Zhou 1991a). This modified virtual
force system is termed adjoint system in the COC methodol-
ogy. A disadvantage of optimality criteria in the continuum
form is that they are not suitable for engineering structures
of a discretized nature, and also the adjoint stress-strain rel-
ations can be complicated to implement in the FE analysis.

In this paper, discretized optimality criteria methods for
structural systems subject to stress constraints and multiple
displacement constraints under multiple load conditions are
developed on the basis of the fundamental principles of the
COC methodology. For historical reasons, we term this ap-
proach the DCOC method in order to distinguish it from the
DOC methods. General optimality criteria for discretized
structural systems are derived on the basis of the flexibili-
ity formulation of matrix analysis and Kuhn-Tucker optimality
condition. Then, an iterative algorithm for coupling the
DCOC method with FE analysis is developed.

For a better understanding of the DCOC algorithm, we
can compare it with the very popular semi-intuitive DOC-
FSD method, in which the displacement constraints are
handled by rigorous optimality criteria but the fully stressed
design concept is used for stress constraints. The iterative
procedure for the updating phase of the DCOC method is ex-
actly the same as that for the DOC-FSD method. However,
rigorous optimality of the DCOC solution is ensured through
modifying the virtual load systems for the displacement con-
straints, by applying initial displacements in members where
stress constraints are active.

Table 1 shows the quantities influencing the computer
time requirement of the primal MP, dual or DOC, DOC-FSD
and DCOC methods in optimizing structural systems with
static behavioural constraints where N is the number of de-
sign variables, m_d is the number of displacement constraints
and m_s is the number of stress constraints. For large struc-
tural systems, the number of active displacement constraints
is usually much smaller than the number of stress constraints,
which results in a very high efficiency of the DCOC method
for such systems. The above contention will also be verified
by numerical examples.

With regard to formulation and derivation of optimal-
ity criteria, the main differences between the DOC and
COC/DCOC methods are summarized in Table 2.

In order to demonstrate the fundamental features of the
DCOC method, optimality criteria are derived for stress con-
straints and a single displacement constraint in Section 2 and