Some aspects of the genesis of structures*

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Abstract Bendsoe and Kikuchi (1988) introduced a novel approach of distributing mass within a specified design domain utilizing a stiffness-density relation obtained by homogenization of a cellular microstructure. This approach was extended to multiple loading cases and three-dimensional applications by the author and his associates (Mlejnek and Schirrmacher 1989; Mlejnek 1990). Moreover the well-known concept of explicit convex behaviour approximation together with a dual solution scheme (Fleury and Smaoui 1988) was successfully introduced to this problem. Further objectives such as eigenvalues and displacements generalized the range of application. It is the aim of this paper, to develop a simplified procedure that can be easily integrated into a FEM-analysis package. Its application requires essentially not much more than the usual FEM-technology. Nevertheless a traceable mathematical base is still maintained. A series of examples demonstrates the suitability of this approach to the preliminary design of minimal compliance structures made of isotropic materials.

1 Introduction
Following Bendsoe’s idea, a preselected design space is discretized and the element densities form the variables of a topological design (Fig. 1). A specified amount of mass, expressed as mean density $\alpha$, is assigned to this design domain and will be redistributed. This task can be treated easily as a sizing problem and frees the design process from almost all such constraints as preselecting movable boundaries, mesh adaptations and initiations of cavities. It involves, however, a large number of variables. Bendsoe and Kikuchi (1988) minimized the compliance for a specified amount of structural mass and constrained density values. This objective leads to an optimization problem, which can be easily handled. In many cases minimal compliance structures form very useful starting designs for other purposes as well. It was therefore decided to restrict the scope of this paper to a compliance objective, but to take into account also multiple loading cases as discussed in previous papers (Mlejnek and Schirrmacher 1989; Mlejnek 1990). One of the main features of this development was to employ standard data input and output of the FEM-package and to add a robust black box optimizer to the main FEM-user program. Thus the information for analysis is more or less also sufficient for preliminary design.

2 Assumptions and mathematical formulation
Bendsoe and Kikuchi (1988) based the density-material stiffness relation on a microstructure model and used homoge-
Standard FEM Preprocessing (e.g. PATRAN)

FEM User Programm OM (e.g. ASKA)

Redesign Loop

Static Analysis
Element Compl.

Effective material data

Optimizer (SCP)

el. compl.

Standard FEM Graphic Postprocessing

Fig. 3. Software scheme

Modulus of elasticity $E_1 = 2.1 \times 10^9$

Poisson's ratio $\nu = 0.3$

Mean density $\alpha = 0.3$

Minimum density $\epsilon_m = 0.01$

Uniform start distribution

Fig. 4. Cantilever under tension and shear: problem

of regions, namely solid, empty and porous regions (SEP-solutions). It was pointed out by the above authors that these solutions are unpractical for several reasons and therefore it is more useful to aim at solutions with only solid or empty elements (SE-solutions). For this latter class of problems, any range of microstructures can be used that includes solid and empty elements as limiting cases. Using solid isotropic microstructures with penalty (SIMP) for intermediate densities has three significant advantages, namely (i) simplicity of analysis and optimization, (ii) selective suppression of porous regions and (iii) capability of handling design conditions other than a single compliance objective (or constraint). The stiffness-density relation suggested in (1) herein is a special case of the SIMP formulation advocated by Rozvany, Zhou and Birker (1992).

We proceed now to the mathematical formulation of our problem. The objective is

$$F = \max_{\ell=1,n} \frac{W_{\ell}}{W_0},$$

$\approx \frac{1}{\rho} \ln \left( \sum_{\ell=1}^{n_f} \exp \frac{W_{\ell}}{W_0} \right) = \min!$ (2)

Here we make use of the Kreisselmeier-Steinhauser approximation of the max-function (Kreisselmeier and Steinhauser 1979), where $W_{\ell}$ denotes the compliance for loading case $\ell$.