Local Boundedness of Minimizers of Integrals of The Calculus of Variations (*).

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Abstract. - In this paper we prove the local boundedness of minimizers of integral functionals with non-standard growth conditions.

0. - Introduction.

This paper deals with local boundedness of minimizers of variational functionals with non standard growth conditions.

Let us consider an integral of the Calculus of Variations of the following type:

\[ J(u) = \int_{\Omega} F(x, u, Du) \, dx, \]

where \( F: (x, u, \xi) \in \Omega \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R} \) is convex with respect to \( \xi \). Let us suppose that there exist two constants \( c_1, c_2 > 0 \) and \( p > 1 \) such that:

\[ F(x, u, \xi) \geq c_1 |\xi|^p - c_2, \quad \text{for every } (x, u, \xi) \in \Omega \times \mathbb{R} \times \mathbb{R}^n. \]

The weakly semicontinuity and the coerciveness of \( J(u) \) in \( W^{1,p}(\Omega) \) ensure that Dirichlet’s problem associated to \( J \) attains its minimum.

Let us assume that:

\[ c_1 |\xi|^p - c_2 \leq F(x, u, \xi) \leq c_3 |\xi|^q + c_4, \quad \text{for every } (x, u, \xi) \in \Omega \times \mathbb{R} \times \mathbb{R}^n. \]

If \( p = q \) i.e. \( F \) satisfies the «natural growth conditions», there are many classical regularity results for minimizers of \( J \) (see, for example,[3],[4],[6]).

The problem of regularity for functionals satisfying «non standard growth conditions», \( p < q \), has been studied extensively in the last few years. In this direction

there are some recent papers of P. Marcellini [7], [8], [9] (see also [2], [10], [12], [14] and their references).

In general, $p$ and $q$ need to be related in some way to obtain results of regularity. For example, if $F(x, u, \xi) = f(|\xi|)$, the condition

$$q < p^* = \frac{np}{n - p}$$

assures locally bounded minimizers (see [2], [10], [12]). Similarly, if $q < \frac{pn}{n - 2}$ the local Lipschitz-continuity of minimizers can be obtained ([7], [8], [9]).

As a further example, let:

$$J_1(u) = \int_\Omega f_1(|Du|) \, dx,$$

where $f_1(t) = t^{a + b \sin(\log t)}$, $t > 1$. This functional $J_1$ has been studied in [2], [9]. It is easy to check that $f_1$ satisfies (0.3) with $p < q$ and, for general values of $a$ and $b$, $p^*$ is allowed to be less than $q$.

Let $F = F(x, \xi)$ be a $C^1$ nonnegative real valued function, convex with respect to $\xi \in \mathbb{R}^n$. Let us define:

$$J_{\lambda}(u) = \int_\Omega (1 + |Du(x)|^2) \frac{\lambda}{2} \, dx,$$

where $\lambda \in L^\infty$. This functional was considered by ZhiKov [15] in a rather different context and