The Discrete Specification of the State-Adjustment Model when the State is Unobservable\footnote{I am indebted to P. Simmons, R. Anderson, L. Phlips and an anonymous referee for helpful comments on an earlier draft. It is unlikely that they agree to share the responsibility for errors and omissions.}

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Summary: This note investigates various ways of specifying the state-adjustment model. It is shown that the value of the structural parameters depends on the discrete specification although the estimating form remains unchanged. Consequently, it is important that analysis involving those parameters be carried out using estimates obtained in a mutually consistent specification of the model; failures to do so may result in erroneous conclusions.

1. The State-Adjustment Model

For the purpose of a presentation in general terms, we introduce the model in continuous time. Since the interpretation of parameters differs in a continuous and in a discrete time model – because of the fixed lags introduced in the latter –, we shall denote parameters of the continuous time model by a star to avoid confusion.

The model consists of one “behavioural equation”:

\[ q(t) = \alpha + \beta s(t) + \gamma y(t), \quad (1) \]

and one “state equation”:

\[ \dot{s}(t) = (1 - \mu) q(t) - \nu s(t). \quad (2) \]

The basic assumption – equation (1) – is that current demand for a commodity, \( q(t) \), depends on its stock (state), \( s(t) \), and current income (total expenditure), \( y(t) \), other variables such as prices being omitted for simplicity. The state can be interpreted as the psychological stock of habits, mainly for non durables, or the physical stock of durables. For the latter \( \beta^* \) is expected to be negative while in the case of habit formation \( \beta^* \) has to be positive.
The evolution of the state is determined by (2) where $v^*$ is a constant depreciation rate. The factor $(1 - \mu^*)$ is rationalized on the grounds that, on the one hand, durables are depreciated once bought and, on the other hand, only a proportion of new flows may enter the stock of habits.

As pointed out by Winder [1971] and Weiserbs [1972], the system (1) -- (2) can be presented in the form of the standard stock-adjustment model:

$$s'(t) = \phi^*[\hat{s}(t) - s(t)],$$

(3)

the tilde denoting a long-run equilibrium value. The long-run is defined by the constancy of all variables so that $s = 0$ and

$$\hat{q}(t) = \frac{v^*}{1 - \mu^*}\hat{s}(t).$$

Since, from (1)

$$\hat{q}(t) = \alpha^* + \beta^*\hat{s}(t) + \gamma^*y(t),$$

it then follows

$$s(t) = \frac{1 - \mu^*}{v^* - \beta^*(1 - \mu^*)}[\alpha^* + \gamma^*y(t)],$$

(4)

which substituted in (3) yields

$$\phi^* = v^* - \beta^*(1 - \mu^*).$$

(5)

The system is stable, provided that $\phi^*$ is positive. This condition may be interpreted as an upper bound to the effect of habit formation.

From (1) and (4), one can easily deduce the short-run and the long-run derivatives with respect to the exogeneous variable:

$$\frac{\partial q}{\partial y} = \gamma^*;$$

(6a)

$$\frac{\partial \hat{q}}{\partial y} = \gamma^* = \frac{\gamma^*v^*}{\phi^*}. \quad (6b)$$

The stock-adjustment model has been introduced in the literature to explain demand for durables. The presentation in terms of a state-adjustment applicable to non durables as well is due to Houthakker/Taylor [1970] [in short H–T].

Most of the time, if not always, the state variable, including its initial value, is unobservable. The usual strategy has been to start with an ad-hoc expression of the system (1) -- (2) in discrete time (i.e. not necessarily a discrete approximation); then to eliminate the state variable by substituting the discrete equivalent of (1) and its first

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3) The reader may observe $\mu$ in trying to trade in a car the same day he bought it.