Spectral Analysis of a Medium Sized German Econometric Model

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A comment to
"J. Wolters: Stochastic properties of a linear econometric model of the
Federal Republic of Germany"

In the above paper an application of spectral analysis is reported for a German econometric model, linear in growth rates,

\[ Ay_t = By_{t-1} + Cx_t + u_t, \quad y_0 \text{ given} \]  

(1)

(A, B, C are coefficient matrices of dimensions \( n \times n \), \( n \times n \), respectively \( n \times m \), and \( y, x, u \) are vectors of endogenous, exogenous, respectively stochastic variables of length \( n, m \), respectively \( n \)) for which the present author has been directly responsible [see Uebe, 1974 and footnote 1 of Wolters]. A number of remarks necessary with respect to the conclusion of J. Wolters:

(i) Alternative estimates of (1) do exhibit a most diverse dynamic behavior.
(ii) The common property of all alternatives is a long swing behavior totally unrelated to the underlying economy.

1. Conclusion (i) is a confirmation of our results derived by deterministic arguments [Uebe, 1974, p. 5].

A third view of the same phenomenon which explains (i) in a most straightforward manner, is to look at the spectra of the underlying time series. Since the intention is not to replicate the above work, yet to elucidate the conclusions, consider e.g. the time series of the GNP [compare figure 1 of Wolters].

Let \( \{Y_t\} \ t = 1, 2, \ldots, 16 \ (t = 1 = 1960, t = 16 = 1975) \) denote the the GNP series (see below the appendix).

Then the spectrum of

\[ \{g_t\} \ t = 1, 2, \ldots, 15 \ (t = 1 = 1961, \ t = 15 = 1975) \]
is determined\(^3\), where \( g_t \) is defined by

\[
y_{t-1} := \left( Y_t - Y_{t-1} \right) / Y_{t-1}, \quad t = 2, 3, \ldots, 16
\]

\[
\tilde{y} = \sum_{t=1}^{15} y_t / 15
\]

\[
g_t := y_t - \tilde{y}, \quad t = 1, 2, \ldots, 15
\]

Imposing on \( g_t \) a fraction of the sample variance of \( y \) by looking up a pseudorandom number generator\(^4\), a sample of 1500 (i.e. 100 simulations) observations is generated:

\[
\tilde{g}_t = g_t + \tilde{u}_t \quad (t = 1, 2, \ldots, (15 \times 100))
\]

where

\[
\tilde{u}_t = p \sqrt{\sigma^2} \epsilon_t, \quad 0 < p \leq 1, \quad \sigma^2 = \sum_{t=1}^{15} g_t^2 / 15
\]

\[
\epsilon_t \sim N(0, 1)
\]

For the sample the spectrum is calculated using an (approximate) Parzen window. A typical spectrum\(^5\) is the following one (see next page) which is distinguished by two properties

a) the spectrum intensities are rather small
b) the spectrum has multiple peaks, e.g. at points 5, 9, 14, 18, 22, 26, 31, where each point \( j \) is at 0 and at \( \frac{j - 1}{32} 2\pi \), \( j \in \{2, 3, 4, \ldots, 33\} \)

Reconsidering the different versions of the model (1) [see e.g. figure 1 of Wolters] it is obvious that none of these versions capture all these empirical peaks. Each is a different smoothing operation on the underlying series. The multipeakedness and the smoothing is in line with what we know from other models (see e.g. Howrey in Hickman [1972, p. 618, 639]) for the multipeaks, and see e.g. the models of both Hickman volumes, which illustrate the smoothing of econometric time series by estimation [Hickman].

\(^3\) Since so much depends on the transformations, each step is put forward in detail, see e.g. Parzen in Hickman [1972, p. 669].

\(^4\) The random number generator used is SUBROUTINE RANNOR (0) of the Leibniz Computer Center, Munich.

\(^5\) The spectrum subroutine is the IMSL SUBROUTINE FTFFT1.