Price Oligopoly as a Cooperative Game

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Abstract: We consider an oligopolistic market as follows. In the market, one good is traded for money. Each oligopolist is a price setter and has the same linear cost function. Each buyer is a price taker and buys the good from oligopolists setting the lowest price. We formulate this market as a cooperative game, and consider two kinds of solution concepts, the core and a bargaining set of the game. First we show that in the monopolistic market, the core gives the monopoly price, but in the oligopolistic market, the core is empty. Second, we obtain the bargaining set of the oligopolistic market.

1. Introduction

It is commonly recognized that in oligopolistic markets, cooperative behavior of oligopolists like joint profit maximization plays a considerable role. This point has been argued by many authors [e.g. Chamberlin; Fellner], though most of them are not formal. The purpose of this paper is to consider such a problem by formulating an oligopolistic market as a cooperative game.

We consider a market as follows. In the market, one kind of good is traded for money. The set of all agents consists of a finite number of oligopolists and a continuum of buyers. Each oligopolist is a price setter and has the same linear cost function. Each buyer is a price taker and buys the good from oligopolists setting the lowest price. Then this market is formulated as a cooperative game.

We consider two kinds of solution concepts, the core and a bargaining set of the game. The core of the monopolistic market coincides with the solution of the classical theory of monopoly. But the core of the oligopolistic market (i.e., that with more than one sellers) is empty. The reason for this is that the rule of the game gives the power to decide a price to the coalition of the oligopolists. The bargaining set gives an interesting version. The bargaining set of the monopolistic market coincides with the core, i.e., the classical solution for monopoly. In the oligopolistic market, the highest price in the bargaining set is decreasing and approaches the price giving half of the maximum joint profit as the number of the oligopolists becomes large. The lowest

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price in it is also decreasing and approaches the competitive price as the number of the oligopolists becomes large.

The bargaining set employed in this paper is a modification of the original ones of Aumann/Maschler [1964] and the one of Peleg [1969].

In section 2 the market model is presented and some lemmas are given, which are necessary to formulate the market. In section 3 the game is defined and the core is considered. In section 4 we obtain the bargaining set. In section 5 we discuss the market model, the game and the results.

2. The Market Model

Let \((I, \bar{I}, \mu)\) be a measure space of all economic agents. \(I = T \cup N = [-1, 0]\)
\(\cup \{1, \ldots, n\}\) denotes the set of all agents, where the closed interval \(T\) is the set of all buyers and \(1, \ldots, n\) are oligopolists. \(I\) is the minimal \(\sigma\)-field which contains the Borel measurable sets and \(\{1\}, \ldots, \{n\}\). \(\mu\) is the \(\sigma\)-additive measure on \(I\) such that \(\mu\) coincides with the usual Lebesgue measure on \(T\) and \(\mu(\{1\}) = \ldots = \mu(\{n\}) = 1/n\). A set in \(I\) is called a coalition.

All the oligopolists sell the same homogeneous good, and have the same linear cost functions:

\[ C(Q) = cQ \quad \forall Q \geq 0. \]

Here \(c\) is a positive real number. We assume that there is no capacity limit.

Each buyer \(t \in T\) has a utility function \(u_t(q)\) which is measured in terms of money.\(^3\) \(u_t(q)\) is defined on the set of all nonnegative real numbers. We assume the following conditions:

(2) \(u_t(q)\) is monotonically increasing, continuous and strictly concave,

(3) \(u_t(q)\) is uniformly bounded from above, i.e., there is an \(m\) such that 
\[ \forall t \in T, \ u_t(q) < m \ \forall q \geq 0, \]

(4) \(u_t(q)\) is measurable with respect to \(t\), i.e., for any \(q\) and \(x\), the set 
\[ \{t \in T \mid u_t(q) > x\} \] is in \(I\).

Here we can put \(u_t(0) = 0 \ \forall t \in T\) without loss of generality.

We define the behavior of oligopolists and buyers as follows. Each oligopolist is a price setter, i.e., he can decide a price at which he sells the good. He produces the quantity of the good which is demanded by buyers.\(^4\) Each buyer is a price taker and buys the good from oligopolists setting the lowest price.

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\(^3\) This ‘transferable utility’ assumption is made just for simplicity. It can be replaced by other adequate assumptions.

\(^4\) Instead of this, we may assume that each oligopolist has a sufficient quantity of the good in stock.