Bounce Problem with Weak Hypotheses of Regularity (*)

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Summary. - In this paper the elastic bounce problem is formulated in very general hypotheses. More precisely we consider the motion of a material point constrained to move in a domain \( \Omega \subset \mathbb{R}^n \), bouncing against its boundary, and we suppose that \( \Omega \) is neither regular nor convex. Assuming that \( \Omega \) is in the class of \( p \)-convex sets introduced in [4] and \( \partial \Omega \in C^{0,1} \), an existence theorem is stated.

Introduction.

In several recent papers the elastic bounce problem for a material point constrained to move in a bounded regular domain \( \Omega \subset \mathbb{R}^n \) elastically reflected by the boundary of \( \Omega \) (see [1], [6], [7]) has been studied. We remark explicitly that a function \( x: [0, 1] \rightarrow \mathbb{R}^n, x \in \text{Lip} ([0, 1]) \) is said to be a solution of the elastic bounce problem if, setting \( \Omega = \{ x: f(x) < 0 \} \) with \( f \in C^1 \) and \( df(x) \neq 0 \) on \( \partial \Omega \), we have

i) \( f(x(t)) < 0 \);

ii) there exists a positive Radon measure \( \mu \) on \([0, 1]\) such that

\[
\text{spt } \mu \subset \{ t: f(x(t)) = 0 \} \quad \text{and} \quad \ddot{x} = -\mu \nabla f(x(t));
\]

iii) the function \( \delta: t \rightarrow |\dot{x}(t)|^2 \) is continuous on \([0, 1]\).

Indeed such a formulation applies only when \( \Omega \) is regular enough (namely \( \partial \Omega \in C^1 \)) and we have existence results only when \( f \) is at least of class \( C^{1,1} \).

Another formulation was given by M. SCHATZMAN assuming the convexity of \( \Omega \) and neglecting the regularity of \( \partial \Omega \) (see [8]).

In both cases we cannot consider—for example—those domains of \( \mathbb{R}^n \) which are piecewise «regular» and piecewise «convex» (see Fig. 1).

The aim of this paper is to find an adequate formulation of the elastic bounce problem (which is equivalent to the preceding ones in the «regular» and in the «convex» case) and a larger class of domains (in general neither regular nor convex) to which such a formulation can be applied.

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Here is an outline of the paper.

In Section I we give some definitions and results which are useful tools in proving our main theorems. The notion of \((p, q)\)-convexity is defined and its connection with \(\Gamma\)-convergence is pointed out.

In Section II we reformulate the elastic bounce problem in non convex and non regular domains and we state an "equivalence" theorem between the "classical" and this new formulation of the elastic bounce problem; moreover an existence theorem in \(p\)-convex domains having Lipschitz continuous boundary is stated.

Sections III and IV are devoted to the proof of the previous theorems; as for the latter case we remark that the method consists in defining suitable approximating problems whose limit (in the sense of \(\Gamma\)-convergence) is the generalized elastic bounce problem itself.

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I. – Notations and preliminary results.

Let \((X, d)\) be a metric space and \(f_h: X \to \mathbb{R}\). We set

\[
\Gamma(X^-) \liminf_{h \to \infty} f_h(x) = f_m(x) = \sup_{A \in \mathcal{D}_x} \liminf_{h \to \infty} f_h(y)
\]

\[
\Gamma(X^-) \limsup_{h \to \infty} f_h(x) = f^*_m(x) = \sup_{A \in \mathcal{D}_x} \limsup_{h \to \infty} f_h(y)
\]

and we say that

\[
\Gamma(X^-) \lim_{h \to \infty} f_h(x) = f_m(x)
\]

if and only if \(f^*_m(x) = f_m(x) = f_m(x)\).