Games with Multiple Payoffs

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Abstract: The traditional theories of decision making and games are based on an assumption that prevents their broader practical utilization: a single dimensional payoff. In reality, any alternative is likely to imply more than one payoff, e.g. not only costs but also time, price, quality, safety, maintainability, productivity, etc. Similarly, the Theory of Games faces difficulties as we attempt to apply it to the conflict situations of the social and business environment. The assumption that a single dimensional payoff, like money, points or chips, is a realistic consequence of individual moves or strategies, is difficult to sustain. In this short paper, concepts of nondominated solutions and a decomposition of parametric spaces are used to formulate and resolve decision problems with vector payoffs. Both Games against Nature and Two-Person, Zero-Sum frameworks are considered. A concept of compromise solutions is introduced to help the decision maker with further reduction of the nondominated set. Some numerical examples are given.

Introduction

In evaluating a set of decision alternatives we should not abstract from the fact that multiple outcomes or payoffs are natural consequences of actions taken. Such multiplicity of outcomes, generated by different alternatives, has been emphasized by Starr [1971]. A few examples will demonstrate his observations:

- consider alternatives representing available modes of transport (like Super-sonic Transport (SST), Vertical Take-off (VTO), Rapid Transit, etc.) to be evaluated in terms of cost of development, economy of operation, cost of maintenance, noise level, safety, etc.
- drugs are usually administered at different dosage levels (alternatives) to different groups of users. Their effects on blood pressure, respiratory efficacy, urinal chemistry, etc. will be different and varied.
- we are to choose a proper machine for performing some production task. We could be concerned about available machines’ productivity, maintainability, set-up costs and times, price, number of rejects, etc.
even if we are interested in a single criterion only, like profitability of a given venture, we might consider its level after one, five, and ten years — different outcomes. Multiple outcomes can be measured in dollars, points, degrees, numbers, etc., and thus multiple payoffs result. If we could combine all such payoffs into a single measure (like total utility) then the alternative with the highest level of utility would be chosen and there would be no decision problem (or just a very trivial one). Rather, we would face a problem of utility measurement.

If we do not know how to measure "utility" or do not wish to lose informational richness offered by the multiplicity of outcomes, we have a problem of Decision Making With Multiple Payoffs.

Traditional decision-making tools, like Games against Nature or Two-person Games, are based on the concept of a single payoff (gain, regret, utility). The Theory of Games, modelled after the simple zero-sum games people play, does not capture full complexity of the real conflict situations. When two opponents choose their strategies (alternatives), not only does the result exhibit a non-zero sum, but it also takes a form of a vector rather than of a scalar. Real games are rather like both players winning some blue chips and losing some red chips after each move. There are no immediate winners or losers; the usage of mixed or randomized strategies is inadequate; and cooperation often replaces competition.

Historical Note

It appears that vector payoffs were first considered by Blackwell [1956] and later by Contini [1966]. Both writers formalized the problem with its full stochastic ramifications. A vector of payoffs, say $c = (c_1, c_2, \ldots, c_t)$ consists of stochastic variables, i.e. a joint probability distribution function can be defined, say

$$f_{ij}(c_1, c_2, \ldots, c_t) \text{ or } f_{ij}(c),$$

for each pair of pure strategies of two players, $i$ and $j$. Instead of having a scalar in the cell $(i, j)$ of the payoff matrix, there is now a more complex situation, as for example in Figure 1.

Depending on the circumstances, from (1) we could either derive marginal distributions for each separate random payoff, or, assuming their independence, derive the joint distribution from them. As both players make their move, a payoff $c$ is selected according to $f_{ij}$. Blackwell considers the case when both players can perform a large number of such moves in a sequential mode. The problem is: "Is there a set $S$ in an $l$-space such that the value of the game, in a long series of plays, is in or arbitrarily close to $S$, with probability approaching 1 as the number of plays becomes infinite?"

Such concept of approachability, although sound for sequential games, is not useful in decision making where we "play" just once and mixed strategies are thus difficult to implement. An approachable set should be replaced by a nondominated set in such situations.