The NMR sensitivity achievable with a slotted-tube resonator

Norio Iriguchi*, Satoshi Yamai and Jun Hasegawa

Siemens-Asahi Medical Technologies Limited, Tokyo 141, Japan

The signal-to-noise ratio achievable with the slotted-tube resonator (STR), a fundamental type of high radiofrequency coil for nuclear magnetic resonance (NMR) experiments, was formulated in an equation. This equation is based on formulae presented by Hour and Richards [J Magn Reson 24, 71 (1976)] and Hoult and Lauterbur [J Magn Reson 34, 425 (1979)]. The equation assumes that the sample is positioned within the homogeneous region of magnetic flux $B_1$ generated by the STR, and involves no unknown factors. The NMR sensitivity of an experiment with an STR can therefore be predicted, and the equation is applicable to any nuclear species, static field strength, and dimensions of the sample and STR.

Keywords: nuclear magnetic resonance (NMR), slotted-tube resonator (STR), signal-to-noise ratio.

INTRODUCTION

Nuclear magnetic resonance (NMR) imaging and spectroscopy are applied in various fields of biomedicine as important diagnostic and analytical techniques. When building a radiofrequency (RF) coil for NMR experiments, it is important to know the NMR sensitivity achievable with the built coil. In particular, when the feasibility of detecting NMR signals is not known for some reason, it is often important to theoretically predict the sensitivity. The NMR sensitivity achievable with conventional coil designs like a solenoid or saddle-shaped coil can be predicted by following the works of Hoult and Richards [1] and of Hoult and Lauterbur [2]. Since conventional coils for whole-body experiments have a relatively low self-resonance frequency, the use of these conventional coils is restricted to the low-frequency range, which is often illustrated by the few pF capacitor required to resonate the coil at a high frequency. Therefore, for high-frequency experiments, low-inductance coil designs like the slotted-tube resonator (STR) have been proposed [3, 4]. The STR is a type of high-frequency coil that is especially useful for physiological or electrically conducting samples. The STR can be modified into a quadrature coil or birdcage design [5], and is one of the most fundamental types of high-frequency coil for whole-body NMR experiments at high frequencies. The electrical circuit equivalent to an STR is known and has been published [6], and the resonant frequency can be determined from the material and dimensional characteristics of the STR. On the other hand, the sensitivity of an STR remains left unknown without conducting an NMR experiment. Therefore, the purpose of this work is to formulate the signal-to-noise ratio (SNR) achievable with an STR by an equation without any unknown factors. The equation in this work is based on the formulae given by Hoult and Richards [1] and by Hoult and Lauterbur [2]. This equation will provide real answers for some important feasibility studies that involve detecting NMR signals, especially when a high magnetic field is required for low-sensitivity nuclear species or when some difficulties are associated with the experiment.

THE SENSITIVITY ACHIEVABLE WITH THE STR

When a unit-volume sample of nuclear spins is exposed to RF flux $B_1$, that is generated by an STR carrying unit current, the electromotive force (EMF) induced in the same STR by processing nuclei in the volume is given by [1] $\xi = - (\delta / \delta t) (B_1 \cdot M_0)$. The unit-
volume nuclear magnetization is \( M_0 = n\gamma^2\hbar^2 I(I+1)B_0/(3kT_s) \), where \( n \) is the number of spins at resonance per unit volume, \( \gamma \) is the magnetogyric ratio, \( I \) is the spin quantum number, \( k \) is Boltzmann's constant, and \( T_s \) is the temperature of the sample. Factor \( n \) is given by \( n = 10^3\nu cA \), where \( \nu \) is the volume of interest in \( m^3 \), \( c \) is the concentration of nuclei of interest in \( mol^{-1} \), and \( A \) is Avogadro's number. When the nuclear spins in the sample have been subjected to a resonant 90° RF pulse, the peak signal EMF is \( \xi_s = 10^3\omega_0\nu cA\gamma^2I(I+1)B_1/(3kT_s) \), where \( \omega_0 \) is the Larmor angular frequency.

An STR is depicted in Fig. 1 and consists of two ring conductors of metal foil mounted on a resin tube (not shown), two vertical conductors of metal foil, and insulating sheets capacitively interconnecting the rings and vertical conductors.

The field strength \( B_1 \) of a saddle-shaped coil carrying unit current is given by the equation of Hoult and Richards [1]. When the subtended angle \( \Omega \) of the vertical conductors is 94° for the best homogeneity [Carlson [7], Fig. 6], the STR produces RF flux \( B_1 \) which is equivalent to that produced by a one-turn, saddle-shaped coil with a subtended wire angle of 60° and the same dimensions, but carrying half the unit current. Therefore, the field strength \( B_1 \) of an STR having diameter \( 2a \) and effective length \( 2l_e \) and carrying unit current can be represented by

\[
B_1 = \frac{3^{3/2}\mu_0}{2\pi}\left\{ \frac{al_e}{[a^2 + l_e^2]^{3/2}} + \frac{l_e}{a[a^2 + l_e^2]^{1/2}} \right\}
\]

where \( l_e = l + \omega/2 \). Specifically, when \( a = l_e \),

\[
B_1 = 0.293\mu_0/a
\]  

(2)

and the EMF induced in the STR is given by

\[
\xi_s = (0.293 \times 10^3\mu_0 Ah^2/3k) x [\varepsilon co_0^2\gamma I(I+1)/T_s a]
\]

(3)

The rms thermal noise EMF is given by \( \xi_N = (4kT\Delta f R)^{1/2} \), where \( \Delta f \) is the bandwidth of the receiver and \( R \) is the effective resistance at temperature \( T \).

From measured values for capacitance \( C_0 \) and electrical resonance frequency \( f_0 \), inductance \( L \) of the STR can be determined. From the measured value for unloaded quality factor \( Q \) of the STR and the calculated \( L \) value, resistance \( R_c \) of the STR can be determined.

When a sample is electrically conductive, noise is caused by thermally generated, randomly fluctuating currents in the sample which are picked up by the STR. The inductive resistance, \( R_i \), of a spherical sample placed in a saddle-shaped coil has been published in an equation given by Hoult and Lauterbur [2]. Regarding the STR as a one-turn, saddle-shaped coil having the same dimensions and carrying half the unit current, the inductive resistance of a spherical sample of diameter \( 2b \) positioned in an STR with radius \( a = l_e \) is given by

\[
R_i = 0.0114\pi\varepsilon o^2 o_0^2 b^3/a^2
\]

(4)

In Eq. 4, \( \varepsilon \) is the specific conductivity of the sample in \( S^{-1} m^{-1} \). The specific conductivity of physiological tissues varies with frequency, the tissue conductivity at a frequency of several 100 MHz being 1.5 to 2.0 times higher than that at several 10 MHz [8].

It is well known that the dielectric losses associated with the coil and sample system are particularly low in an STR [2, 6], and that the noise contribution from the dielectric loss can be neglected.

The rms thermal noise EMF per unit bandwidth induced in the STR is given by \( \xi_N = (4k(T_c + R_i + T_s R)\Delta f)^{1/2} \). The coil resistance for a given geometry and number of turns is independent of the coil size, but increases as the reciprocal of the skin depth [1] leading to the simple relationship \( R_c \propto \omega_0/2 \). So we may define a constant of proportionality \( R^* \) such that \( R_c = R^*\omega_0/2 \). Similarly, the inductive resistance is \( R_i \propto \omega_0^2\varepsilon o^2 b^3/a^2 \), giving \( R_i = R^*\omega_0^2\varepsilon o^2 b^3/a^2 \). Substituting these expressions into the equation for the rms thermal noise EMF \( \xi_N \) per unit bandwidth:

\[
\xi_N = (4kia^2)^{1/2} \times (T_c R^*\omega_0^2 a^2 + T_s R_i^*\omega_0^2 b^3/a^2)^{1/2}
\]

(5)

By combining the foregoing results, and separating the fundamental constants, we arrive at SNR \( \Psi^* \) at

MAGMA (1993) 1(3 & 4)