Reflection and Transmission of Plane Waves by a Layer of Compact Inhomogeneities

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Abstract—Reflection and transmission of elastic wave motion by a layer of compact inhomogeneities has been analyzed. For identical inhomogeneities whose geometrical centers are periodically spaced, the problem has been formulated and solved rigorously. The reflected and transmitted longitudinal and transverse wave motions have been expressed as superpositions of wavemodes, where each wavemode has its own cut-off frequency. At its cut-off frequency a mode converts from a standing into a propagating wavemode. The standing wavemodes decay exponentially with distance to the plane of the centers of the inhomogeneities. At small frequencies only the lowest order modes of longitudinal and transverse wave motion are propagating. Reflection and transmission coefficients have been defined in terms of the coefficients of the zeroth-order scattered wavemodes. These coefficients have been computed by a novel application of the Betti-Rayleigh reciprocal theorem. They are expressed as integrals over the surface of a single inhomogeneity, in terms of the displacements and tractions on the surface of the inhomogeneity. The system of singular integral equations for the surface fields has been solved numerically by the boundary integral equation method. Curves show the reflection and transmission coefficients for the reflected and transmitted longitudinal and transverse waves as functions of the frequency. Some results are also presented for planar distributions of cracks whose spacing and size are random variables. Finally, dispersion relations are discussed for solids which are completely filled with periodically spaced inhomogeneities.

Key words: Reflection, transmission, plane waves, compact inhomogeneities, cracks.

Introduction

The presence of compact inhomogeneities in geological configurations gives rise to a multitude of multiple scattering processes of seismic waves. A mathematical analysis of these scattering processes is feasible for some special cases. These include the case that the compact inhomogeneities are concentrated in a single layer, and a (quasi) ordering of the inhomogeneities can be assumed. This paper reviews some recent mathematical results for such configurations. Examples of the geological structures to which these results may apply, are fault zones, folds consisting of systems of

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quasi-ordered inclusions and fractures, phase transition zones, and layers of regularly spaced sedimentary inclusions. Most of the results discussed here assume a high degree of regularity in the spacing and the sizes, shapes and orientations of the compact inhomogeneities. Some results are, however, also presented for random spacings and random distributions of size parameters, particularly for the case of a planar distribution of cracks.

It is not difficult to show that a periodic array of identical compact inhomogeneities whose geometrical centers are located in a plane interior to an elastic solid, acts as a homogeneous plane of reflection and transmission of incident waves, at least at low frequencies. A typical example is provided by incidence of a plane elastic wave on a single layer of periodically spaced spherical cavities. Reflection and transmission coefficients for that configuration have been obtained by ACHENBACH and KITAHARA (1986). Their results show that for an arbitrary angle of incidence, an incident plane wave gives rise to an infinite number of reflected and transmitted longitudinal and transverse wavemodes. The higher-order modes have cut-off frequencies below which these modes are evanescent. Below the first cut-off frequency only the zeroth-order modes are propagating, and these modes correspond to reflected and transmitted homogeneous plane waves.

The analysis of ACHENBACH and KITAHARA (1986) exploits the periodicity of the array of spherical cavities. Here we generalize that formulation to a periodic array of identical inhomogeneities which are of arbitrary shape. The reflection and transmission problem is formulated rigorously. An infinite number of reflected and transmitted wavemodes is identified, each mode with its own cut-off frequency. Reflection and transmission coefficients have been defined in terms of the coefficients of the zeroth-order wavemodes. These coefficients have been computed by a novel application of the Betti-Rayleigh reciprocal theorem. They are expressed as integrals over the surface of a single inhomogeneity, in terms of the field variables on that surface. The fields on the surface of that inhomogeneity are governed by a system of boundary integral equations which have been derived in some detail. This system has been solved numerically by the use of the boundary element method. Numerical results are presented for spherical cavities and spherical inclusions.

Interaction of an incident wave with a single layer of cracks has been considered by ANGEL and ACHENBACH (1985a,b) for a two-dimensional configuration of equally spaced collinear cracks. A more general formulation for an array of inclined cracks has been provided by MIKATA and ACHENBACH (1988). A brief summary and some results are included in this paper.

Reflection and transmission of wave motion by a more general distribution of coplanar cracks has recently been considered by SOTIROPOULOS and ACHENBACH (1987). This problem, which is of considerable interest for seismological applications, is briefly discussed. In the last section of the paper we briefly comment on the propagation of harmonic waves in a solid which is completely filled with a periodic distribution of inhomogeneities.