Conference Structures and Fair Allocation Rules

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Abstract: To describe how the outcome of a cooperative game might depend on which groups of players hold cooperative planning conferences, we study allocation rules, which are functions mapping conference structures to payoff allocations. An allocation rule is fair if every conference always gives equal benefits to all its members. Any characteristic function game without sidepayments has a unique fair allocation rule. The fair allocation rule also satisfies a balanced contributions formula, and is closely related to Harsanyi's generalized Shapley value for games without sidepayments. If the game is superadditive, then the fair allocation rule also satisfies a stability condition.

Introduction

We expect that players in a cooperative game should meet together in a series of conferences to discuss possible cooperative plans and to sign jointly binding agreements. The goal of cooperative game theory is to help us understand how these conferences determine the ultimate outcome of the game. To accomplish this goal, one could try to model these conferences as dynamic processes or as noncooperative games in their own right; but such models are hard to construct without being ad hoc or unrealistic, because negotiation conferences are in fact very complex phenomena. In this paper, we will try to avoid this difficulty by taking conferences as black boxes. That is, we will study allocation rules, which tell us how the players' payoffs ought to depend on which conferences occur, but we will not try to describe the internal workings of these conferences. We will only assume that the net effect of a conference should be fair, in that it benefits all of its members equally. Our main result is that every game has a unique fair allocation rule satisfying a natural Pareto-efficiency property.

The results of this paper generalize earlier results in Myerson [1977a], by dropping the sidepayments assumption and by allowing for conferences of more than two players. For some other approaches to the question of how final payoffs should depend on the structure of cooperation in a game, see Luce/Raiffa [1957, Chapter 10], Maschler [1963], Aumann/Drèze [1974], Owen [1977], and Shenoy [1979].
1. Basic Definitions

Throughout this paper, we shall let \( V \) be a characteristic function game without sidepayments, and we let

\[ N = \{1, 2, \ldots, n\} \]

represent the set of players in \( V \). That is, we assume that \( V \) is a function which maps each set of players \( S \subseteq N \) onto a set \( V(S) \) such that:

1. \( V(S) \) is a closed subset of \( \mathbb{R}^n \); \hspace{1cm} (1.1)
2. \( \emptyset \neq V(S) \neq \mathbb{R}^n \) if \( S \neq \emptyset \) \( (V(\emptyset) = \mathbb{R}^n) \); \hspace{1cm} (1.2)
3. if \( x \in V(S), y \in \mathbb{R}^n \), and \( y_i < x_i \) \( \forall i \in S \) then \( y \in V(S) \). \hspace{1cm} (1.3)

(Notice that this is somewhat weaker than the usual definition of a characteristic function game without sidepayments, as in Aumann/Peleg [1960]. For example, we shall not need convexity of \( V(S) \) in this paper).

The set \( V(S) \) is interpreted as the set of all payoff allocations which give the members of \( S \) a combination of payoffs which they could guarantee themselves together, without cooperating with the other players. Condition (1.3) asserts that free disposal of utility payoffs is always possible for any coalition \( S \), so that \( V(S) \) must be a comprehensive subset of \( \mathbb{R}^n \).

For any set \( S \subseteq N \), let \( \partial V(S) \) be the weakly Pareto-efficient frontier of \( V(S) \). That is:

\[ \partial V(S) = \{ x \in V(S) \mid \text{if } y_i > x_i \text{ for all } i \in S, \text{ then } y \not\in V(S) \}. \hspace{1cm} (1.4) \]

To describe how the players organize their cooperation, we must specify which groups of players are willing and able to confer together for the purpose of planning cooperative actions. It may be that some players will not talk to each other directly, or that some players can only communicate with each other in the presence of many other players, as in a convention or a committee meeting.

We shall use the term conference to refer to any set of two or more players who might meet together to discuss their cooperative plans. A conference structure is then any collection of conferences. We let \( CS \) denote the set of all possible conference structures, so that:

\[ CS = \{ Q \mid \forall S \in Q, S \subseteq N \text{ and } |S| \geq 2 \}. \hspace{1cm} (1.5) \]

Given a conference structure \( Q \in CS \), players \( i \) and \( j \) are connected by \( Q \) if \( i = j \) or there exists some sequence of conferences \( (S_1, \ldots, S_m) \) such that:

\[ i \in S_1, j \in S_m, \{S_1, \ldots, S_m\} \subseteq Q, \text{ and} \]

\[ S_k \cap S_{k+1} \neq \emptyset \text{ for every } k = 1, \ldots, m - 1. \]

That is, two players are connected by \( Q \) if they can be coordinated either by meeting together in some permissible conference to which they both belong \( (m = 1) \), or by