General Solution of a Nonlinear \((m + n)\)-System
with a Irregular Type Singularity without Poincaré Condition \((*)\) \((**)\).

MASAHIRO IWANO

Summary. – This paper discusses a nonlinear \((m + n)\)-system with an irregular type singularity not satisfying Poincaré condition. By the use of a fixed point technique devised by Prof. Masuo Hukuhara, an \((m + n)\)-parameter family of bounded solutions is constructed. \(D\) is a sectorial domain with vertex at the origin in the complex plane \(C\). The domain of holomorphy for a set of functions appearing in our fixed point technique has to be given in terms of the family of the product of \((m + n)\) discs about \(x\), when \(x\) moves in the domain \(D\). The center of each disc is the origin of \(C\) and its radius depends on not only \(\arg x\) but also \(|x|\).

1. – Introduction.

1. NOTATION. – In order to simplify the description, we introduce some vectorial notation. For an \(l\)-vector \(\lambda\) with entries \(\{\lambda_j\}\), \(1_l(\lambda)\) is a diagonal matrix given by

\[
1_l(\lambda) = \text{diag} \{\lambda_1, \lambda_2, \ldots, \lambda_l\}.
\]

\(-\lambda\) means \(l\)-vector with entries \(\{-\lambda_j\}\), so that

\[
1_l(-\lambda) = \text{diag} \{-\lambda_1, -\lambda_2, \ldots, -\lambda_l\}.
\]

The norm \(\|\lambda\|\) of \(\lambda\) is given by

\[
\|\lambda\| = \max_{j=1}^l |\lambda_j|.
\]

The symbol \(e^l\) denotes the \(l\)-vector with entries \(\{e^j\}\). \(1_l\) means the \(l\)-dimensional unit matrix. When the dimension of a unit-vector is given, \(e_j\) is the unit-vector with vanishing entries except for its \(j\)th-one which is equal to the unity.

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\((**\) Part of this work was done while on visit to the Department of Mathematics and Statistics, Western Michigan University, U.S.A.
For an $l$-tuple $t = (t_1, t_2, \ldots, t_l)$ of nonnegative integers $t_j$, $|t|$, $t\lambda$ and $\lambda^t$ stand for the scalar expressions

$$
\begin{align*}
|t| &= t_1 + t_2 + \ldots + t_l, \\
t\lambda &= t_1\lambda_1 + t_2\lambda_2 + \ldots + t_l\lambda_l, \\
\lambda^t &= \lambda_1^{t_1}\lambda_2^{t_2} \ldots \lambda_l^{t_l}.
\end{align*}
$$

2. ASSUMPTIONS. – We consider two systems of $(m+n)$-nonlinear differential equations of the form

$$
\begin{align*}
x^2 \frac{dy}{dx} &= (1_m(\mu) + x1_m(\alpha))y + f(x, y, z), \\
x^2 \frac{dz}{dx} &= (-1_n(\nu) + x1_n(\beta))z + g(x, y, z).
\end{align*}
$$

We assume that:

(i) $x$ is a complex variable;

(ii) $y$ is an $m$-vector with entries $\{y_j\}$ and $z$ is an $n$-vector with entries $\{z_k\}$; $\mu$ is an $m$-vector with positive real entries $\{\mu_j\}$ and $\nu$ is an $n$-vector with positive real entries $\{\nu_j\}$; $\alpha$ is an $m$-vector with complex entries $\{\alpha_j\}$ and $\beta$ is an $n$-vector with complex entries $\{\beta_j\}$;

(iii) $f(x, y, z)$ and $g(x, y, z)$ are $m$- and $n$-vectors respectively, and their entries are holomorphic and bounded functions of $(x, y, z)$ at the origin of the complex $C^{1+m+n}$-space, namely in a domain of the form

$$
|x| < a, \quad \|y\| < b, \quad \|z\| < b.
$$

The Taylor expansions of the entries of $f$ and $g$ in powers of the entries of $y$ and $z$ begin with terms of degree at least 2:

$$
\begin{align*}
f(x, y, z) &= \sum_{|p| + |q| \geq 2} f_{pq}(x)y^pz^q, \\
g(x, y, z) &= \sum_{|p| + |q| \geq 2} g_{pq}(x)y^pz^q,
\end{align*}
$$

where $p = (p_1, p_2, \ldots, p_m)$ with nonnegative integers $p_j$ and $q = (q_1, q_2, \ldots, q_n)$ with nonnegative integers $q_k$; the coefficients $f_{pq}(x)$ and $g_{pq}(x)$ are $m$- and $n$-vectors and their entries are holomorphic and bounded functions of $x$ for $|x| < a$.

We assume moreover that:

(iv) the $\mu_1, \ldots, \mu_m, \nu_1, \ldots, \nu_n$ are independent over the ring $\mathbb{Z}$ of all integers;

(v) there exists a positive number $\kappa$ such that

$$
\begin{align*}
\mu_j(\kappa) &\equiv \mu_j + \kappa \cdot \Re \alpha_j > 0 \quad \text{(for all $j$)}, \\
\nu_k(\kappa) &\equiv -\nu_k + \kappa \cdot \Re \beta_k > 0 \quad \text{(for all $k$)}.
\end{align*}
$$