CALCULATION OF BEAMS AND SLABS ON AN ELASTIC BASE
WITH CONSIDERATION OF A BREAK IN CONTINUITY AND CREEP
OF THE ELASTIC BASE

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On the basis of previously published works, a solution of problems by the step method is presented (calculation of beams and slabs on an elastic base with consideration of a break in continuity and creep of the base). An example of calculation of a horizontally loaded pile is given.

It is known that cohesive soils possess properties of a viscoelastic body to a significant degree [1]; therefore, in calculating beams and slabs on such soils with consideration of a break in the base’s continuity, one must take into account their creep flow. The calculation is based on S. N. Klepikov’s idea, which he implemented in calculation of statically indeterminate systems by the step method [2]. The essence of his method is that, in each successive step in time \( t_n = t_{n-1} + \Delta t \), it is considered that the extra unknown \( X_i(0) + \Delta X_i(1) + \ldots + \Delta X_i(n-1) \) performs work on elastoviscous displacements; and the increment \( \Delta X_i(n) \), on elastic displacements in the basic system of the work method.

Analysis of results of calculation of beams and slabs on an elastic base with consideration of a break in continuity [3-9] makes it possible to divide this problem into two parts:

1) with an unknown region of contact, where the solution is sought by means of iteration [3-6].
2) with a known region of contact [7-9].

We will consider the first part of the problem, which is harder to solve. We will implement the algorithm in [2] for calculating a horizontally loaded pile (Fig. 1). Solution of this problem without consideration of creep is given in [3]. We will perform the calculation by B. N. Zhemochkin’s method [10]. The basic system of the mixed method for solving the problem is shown in Fig. 2.

We will note that, with the course of time, the forces in B. N. Zhemochkin’s constraints and the position of the zero point \( a \) change (see Fig. 2). In each step in time, we will analyze the signs of forces in B. N. Zhemochkin’s constraints and the shape of the gap created as a result of bending [3].

Step 1. We will perform the usual elastic calculation [3]. To do this, we will write the system of canonical equations for calculating a horizontally loaded pile with consideration of a break in continuity of the elastic base [3] in a more compact form:

\[
\begin{align*}
\sum_{i=1}^{\bar{c}} \left( \sum_{v=1}^{\bar{v}} \left((V_{ik} + \omega_{ik}) X_i^{(0)} - \frac{2l - 1}{2} c\phi^{(0)} - u_0 \right) \right) &= 0; \\
\sum_{k=1}^{\bar{k}} \frac{2k - 1}{2} c X_i^{(0)} &= M; \\
\sum_{k=1}^{\bar{k}} \sum_{v=1}^{\bar{v}} X_i^{(0)} &= P
\end{align*}
\]  

(1)

where \( V_{ik} \) is a term of the matrix of displacements of boundaries of a rectangular gap in the elastic base. The sequence of determining them is given in [3]; \( \omega_{ik} \) is deflection of the point \( i \) of a rod in the basic system of the mixed method from \( X_k = 1 \); and \( X_i^{(0)}, \phi^{(0)}, \) and \( u_0^{(0)} \) are unknown forces and displacements of the top of the rod, determined according to elastic calculation at the initial moment in time \( t_1 = r_1 \).
Step 2. The moment in time \( t_2 = t_1 + \Delta t \). According to [2], during the time \( t_2 - t_1 \), forces \( X_{i}^{(0)} \) perform work on elastoviscous displacements; and the increment \( \Delta X_{i}^{(1)} \), on elastic displacements. The system of canonical equations, with a fixed position of point \( a \) (see Fig. 2), takes the form

\[
\begin{align*}
\sum_{i=1}^{N} \left( \sum_{k=1}^{K} \left( \frac{1}{2} \omega_{ik} + \psi_{01} \right) V_{ik} + \frac{1}{2} \omega_{ik} X_{ik}^{(0)} + \left( V_{ik} + \omega_{ik} \right) \Delta X_{ik}^{(1)} - \frac{2i-1}{2} c \left( \psi_{01}^{(i)} + \Delta \psi_{01}^{(i)} \right) - \frac{2i-1}{2} c \left( \psi_{01}^{(i)} + \Delta \psi_{01}^{(i)} \right) \right) = 0; \\
\sum_{k=1}^{K} \frac{2k-1}{2} c \left( X_{ik}^{(0)} + \Delta X_{ik}^{(1)} \right) = M; \\
\sum_{k=1}^{K} \left( X_{ik}^{(0)} + \Delta X_{ik}^{(1)} \right) = P,
\end{align*}
\]

where \( \psi_{01} \) is the creep coefficient [2].

Comparing Eqs. (1) and (2), after simplifications, we get

\[
\begin{align*}
\sum_{i=1}^{N} \left[ \sum_{k=1}^{K} \left( V_{ik} + \omega_{ik} \right) \Delta X_{ik}^{(1)} - \frac{2i-1}{2} c \Delta \psi_{01}^{(i)} - \frac{2i-1}{2} c \Delta \psi_{01}^{(i)} \right] = S^{(3)}; \\
\sum_{k=1}^{K} \frac{2k-1}{2} c \Delta X_{ik}^{(1)} = 0; \\
\sum_{k=1}^{K} \Delta X_{ik}^{(1)} = 0
\end{align*}
\]