Lagrangean Relaxation and Constraint Generation Procedures for Capacitated Plant Location Problems with Single Sourcing

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Summary. Constraint generation procedures for identifying facet-induced inequalities violated by the optimal solution to the current LP relaxation have been widely used to solve integer programming problems. For capacitated plant location problems, Barcelo has recently tested computationally the performance of one such procedure. Hallefjord and Jörnsten have shown how these procedures can lead to better bounds when used with Lagrangean relaxations instead of the classical LP ones. We describe a Lagrangean relaxation heuristic algorithm for the capacitated location problem that in each iteration expands the dual space by adding to the dual Lagrangean function a new valid inequality for the problem generated from the current partial solution. Examples and computational results are included.


Introduction

Discrete location problems have been extensively studied in the past and several applications have been presented in a number of different areas. For a survey on location problems see [8], [13] and [17].

Among the areas of application we can mention the following:

- The choice of the appropriate size and location of the platforms to be used for the drilling of oil wells. ([2], [9]). In this application the decision variables are
  a) the number of platforms,
  b) the size of each platform,
  c) the location of each platform and
  d) the assignment of wells to platforms.

Depending on the cost structure many different models in the discrete location family can be generated ranging from an uncapacitated plant location model to a capacitated model with a "complex" cost function. See [9] for a detailed presentation.

- Cluster analysis

The standard problem of cluster analysis, that is the problem of grouping data entities in a way as to maximize the homogeneity of points within one group and at the same time the heterogeneity of the points between groups, can be formulated as a problem in the discrete location family.

Lagrangean relaxation based solution methods for the uncapacitated and capacitated cluster analysis problem are presented in [20] and [21].
An application for the determination of strata boundaries for use in stratified sampling is given in [19].

- Routing

Yet another application related to cluster analysis is presented in [11] where the problem is to group customers into vehicle routes such that the total customer requirements do not exceed the delivery capacity of the vehicle(s) assigned to the route(s).

More locational related applications are
- Assignment of cities to garbage dumps [4].
- Location of bank accounts [6].

We can also mention that the generalized bin-packing problem [18] falls into this class of models.

In all the applications mentioned above both the location decision and the assignment decision are modelled using integer (0, 1) variables. This means that it is not possible to split a "customer" among several destinations. In this paper we will consider this special class of pure integer discrete location problems. We will present a new algorithm based on the combination of Lagrangean relaxation techniques and generation of valid inequalities.

In the algorithm we have to solve subproblems that are uncapacitated plant location problems. This can for instance be done by the dual ascent procedure developed by [10].

2. Problem Formulation

We will use the following formulation of the capacitated plant location problem:

\[(P) \min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} f_j y_j\]

subject to

\[\sum_{j \in J} x_{ij} = 1 \quad \forall \ i \in I \quad (1)\]

\[b_j y_j - \sum_{i \in I} d_i x_{ij} \geq 0 \quad \forall \ j \in J \quad (2)\]

\[y_j - x_{ij} \geq 0 \quad \forall \ i \in I, \forall \ j \in J \quad (3)\]

\[\sum_{j \in J} b_j y_j \geq D \quad (4)\]

\[x_{ij} \in \{0, 1\}, y_j \in \{0, 1\} \quad \forall \ i \in I, \forall \ j \in J \quad (5)\]

\[I=\{1, \ldots, n\} \text{ is a set of demand centres and } J=\{1, \ldots, m\} \text{ is a set of potential plant location sites. The variables are defined as}\]

\[y_j =\begin{cases} 1 & \text{if a plant is opened at site } j \\ 0 & \text{otherwise}\end{cases}\]

\[x_{ij} =\begin{cases} 1 & \text{if demand point } i \text{ is assigned to a plant at site } j \\ 0 & \text{otherwise}\end{cases}\]

Other notation:

\[c_{ij} \text{ The cost of assigning demand point } i \text{ to a plant at site } j\]

\[f_j \text{ the cost of opening a plant at site } j \text{ (measured in units comparable to the } c_{ij} \text{’s)}\]

\[b_j \text{ the capacity of plant } j\]

\[d_i \text{ the demand at demand point } i\]

\[D = \sum_{i \in I} d_i, \text{ aggregated demand.}\]

Other definitions of variables and problem data may be appropriate in another context. If we for instance were studying the problem of assigning wells to platforms, \(y_j\) would be defined as

\[y_j =\begin{cases} 1 & \text{if a platform is placed at site } j \\ 0 & \text{otherwise}\end{cases}\]

In the plant location context, constraints (1) imply that each demand point is assigned to one and only one plant. Constraints (2) are the capacity constraints – a plant at site \(j\) cannot deliver more than \(b_j\) units. Constraints (2) also guarantee that customers are served by existing plants only. Constraints (3) and (4) are redundant in this statement of the problem, but they can be useful for developing efficient solution techniques, see for instance [25].

3. Lagrangean Relaxation of Capacity Constraints

By performing a Lagrangean relaxation of constraints (2) and (4), an uncapacitated plant location subproblem is obtained:

\[(L) \min \sum_{i \in I} \sum_{j \in J} (c_{ij} + u_j d_i) x_{ij} + \sum_{j \in J} [f_j - (u_j b_j + v b_j)] y_j + v D\]

subject to