This paper studies the asymptotic behavior of functions $M(n, k, k - i, \lambda)$ and $m(n, k, k - i, \lambda)$, equal to the respective cardinalities of the minimal $\lambda$-covering and maximal $\lambda$-packing of all $(k - 1)$-subsets of the $n$-element set of its $k$-subsets. It is shown that, if sequence $k = k(n)$ is such that $k(n)/n \to 0$ as $n \to \infty$ then $m(n, k, k - 1, \lambda) = o\left(k \cdot \binom{n}{k - 1}\right)/k$, and if $k(n)/\sqrt{n} \to \infty$ as $n \to \infty$, then $M(n, k, k - 1, i) = \Omega\left(\binom{n}{k - 1}\right)/k$.

A consequence of these results is the validity of the Erdős–Hanani conjecture concerning the asymptotic behavior of functions $M(n, k, k - 1, 1)$ and $m(n, k, k - 1, 1)$.

The problem is considered of the covering and packing of subsets of a finite set. Let $S_n$ be an unordered set consisting of $n$ distinct elements. We shall denote by $B^k(S_n)$ the set of all $k$-subsets of $S_n$, i.e., the subsets of cardinality $k$. Let $n \geq k \geq l \geq 1$ be natural numbers, and let $1 \leq \lambda \leq \binom{n - 1}{k - l}$ be a natural number. The system of $k$-subsets $P \subseteq B^k(S_n)$ is called a $\lambda$-tuple covering of set $B^l(S_n)$ if, for any element of set $B^l(S_n)$, there exist no fewer than $\lambda$ distinct elements of $P$ containing it as a subset. We denote by $M(n, k, l, \lambda)$ the cardinality of the smallest possible $\lambda$-tuple covering $P \subseteq B^k(S_n)$ of set $B^l(S_n)$. It should be mentioned that the binomial coefficient $\binom{n - 1}{k - l}$ gives us a limit value of multiplicity $\lambda$ and, therefore, the inequality $\lambda \leq \binom{n - 1}{k - l}$ always holds.

A system of $k$-subsets $Q \subseteq B^k(S_n)$ is called a $\lambda$-fold packing of set $B^l(S_n)$ if, for any element of set $B^l(S_n)$, there exist no more than $\lambda$ distinct elements of $Q$, each of which contains it as a subset. We denote by $m(n, k, l, \lambda)$ the cardinality of the largest possible $\lambda$-fold packing $Q \subseteq B^k(S_n)$ of set $B^l(S_n)$.

The problem amounts to finding the values of the quantities $M$ and $m$ as functions of $n, k, l$, and $\lambda$. In this formulation, the problem was considered in [1]. In [2] the formulation of this problem was treated for the case when $\lambda = 1$. The problem also permits an equivalent formulation in the language of the theory of hypergraphs [3].

Fort and Hedlund [4] proved that

$$M(n, 3, 2, 1) = \lfloor n/3 \rfloor (n - 4)/2,$$

where $\lfloor x \rfloor$ denotes the least integer no smaller than $x$. Schönheim [5] determined the value of $m(n, 3, 2, 1)$. Hanani [6] proved that

$$M(n, 4, 3, 1) = m(n, 4, 3, 1) = \binom{n}{3}/4$$

when $n \equiv 2 \text{ or } 4 \pmod{6}$.

Stanton and Kalbfleisch [7] found that

$$M(n, 4, 3, 1) = \frac{n^3 - 2n^2 + 3n + 6}{24}$$

when $n \equiv 3 \text{ or } 5 \pmod{6}$.
and also that
\[ M(n, 4, 3, 1) = \frac{n(n^2 - 3n + 6)}{24} \] when \( n = 2^m \cdot 6, m = 0, 1, 2, \ldots \).

Mills [8, 9] found the value \( M(n, 4, 2, 1) \). The well-known theorem of Turán [10] gives an exact value for \( M(n, n - 2, l, 1) \). A detailed survey of results for this problem is contained in [11].

It is known (see, e.g., [1, 2]) that
\[ m(n, k, l, \lambda) \ll \frac{n}{\binom{n}{l}} \ll M(n, k, l, \lambda). \] (1)

Since exact values for the quantities \( M(n, k, l, \lambda) \) and \( m(n, k, l, \lambda) \) are known only for particular cases, it is of interest to have an asymptotic solution to the problem.

Erdős and Hanani [2] advanced the conjecture that, for \( k \) and \( l \) not depending on \( n \),
\[ \lim_{n \to \infty} m(n, k, l, 1) \cdot \binom{n}{l} / \binom{n}{k} = \lim_{n \to \infty} M(n, k, l, 1) \cdot \binom{n}{k} / \binom{n}{l} = 1. \] (2)

and proved it for the case \( l = 2 \), and also for \( l = 3 \) and \( k = p + 1 \), where \( p \) is a power of a prime.

In the present paper we shall show that if \( k = o(n) \), then
\[ \lim_{n \to \infty} m(n, k, k - 1, \lambda) \cdot k / \binom{n}{k - 1} = 1, \]
and, if \( k = o(\sqrt{n}) \), then
\[ \lim_{n \to \infty} M(n, k, k - 1, \lambda) \cdot k / \binom{n}{k - 1} = 1. \]

The proof will be based on the connection among packings, coverings, and sets of solutions (of a definite form) of some system of congruences. The use of congruences for solving combinatorial problems appeared earlier (see, e.g., [12-14]).

**THEOREM 1.** If \( k = o(n) \), \( n \to \infty \), \( 1 \leq \lambda \leq n - k + 1 \), then
\[ \lim_{n \to \infty} m(n, k, k - 1, \lambda) \cdot k / \binom{n}{k - 1} = 1. \] (3)

**Proof.** We arbitrarily number the elements of set \( S_n \) by the numbers from \( 1 \) to \( n \). We consider the following congruence (for the definitions and rules for operating with congruences, see [15]):
\[ \sum_{i=1}^{k} x_i \equiv c \pmod{n}, \] (4)
\[ x_i \in \{1, 2, \ldots, n\}, \] \( c \) is a fixed number from \( \{0, 1, \ldots, n - 1\} \),
\[ x_i \neq x_j, \quad i, j = 1, 2, \ldots, k \; (i \neq j). \] (5)

We denote the quantity of unordered solutions to (4) under constraint (5) by \( t(n, k, c) \), and the quantity of ordered solutions, by \( p(n, k, c) \). It is not hard to verify the validity of the following property:
(A) any two unordered solutions to the congruence of (4) have at most \( k - 2 \) identical terms.

We also note that by virtue of (5),
\[ t(n, k, c) = p(n, k, c) / k!. \] (6)

To any unordered solution to congruence (4), with constraint (5), there is a single-valued corresponding selection of those \( k \) elements of set \( S_n \) whose ordinal numbers occur in the given solution. Thus, the congruence gives us some subset \( Q(n, k, c) \subseteq B^c(S_n) \), which is a 1-packing \( B^{k-1}(S_n) \) by virtue of property (A), where