IS THE KOLMOGOROV REGIME THE MOST ORGANIZED HYDRODYNAMIC TURBULENCE STATE?
(A RENORMALIZATION GROUP APPROACH)

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The renormalization group approach to hydrodynamical turbulence is studied from the standpoint of synergetics. We have shown that the Kolmogorov regime can be regarded as a result of self-organization taking place in turbulent media. The renormalization, in turn, can be considered as an initial turbulence state mapping to the final one, which is the result of self-organization.

1. Introduction

Since 1941 when, basing on the velocity field self-similarity hypothesis, \( k^{-5/3} \) turbulence spectrum had been derived by Kolmogorov and Obukhov, many attempts were made to obtain it directly from fluid dynamics basic equations, i.e. from Navier-Stokes equations. Nevertheless, we are to mention that all these theoretical studies, aimed to gain the Kolmogorov spectrum, directly or indirectly contained the inertial range existing hypothesis. Mathematically that means the existing of a domain in \( k \)-space

\[ m \ll k \ll \Lambda, \]

such that Green functions depend only on the total energy income, but not on viscosity. The existing of such domain implies certain scaling relation for Green and response functions.

In present paper we analyse why the inertial range existing hypothesis cannot be proved in the statistical micromodel framework, i.e. directly from Navier-Stokes equations for incompressible viscous fluid driven by random Gaussian force. We argue that the Kolmogorov hypothesis, nevertheless, can be rejected for the sake of a more general synergetic principle. The latter can be formulated as follows: An open dissipative system (hydrodynamics, in particular) tends to the most organized state, allowed by its environment.

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2. Stochastic hydrodynamics as a field theory

The most common hydrodynamic turbulence micromodel is the Navier-Stokes equations system for incompressible viscous fluid driven by random force $f(x)$.

$$ L[\varphi(x)] \equiv \partial_t \varphi + (\vec{\varphi} \vec{\partial}) \vec{\varphi} - \nu \nabla \varphi + \partial p = f(x), \quad (1) $$

where $x \equiv (t, \vec{x})$, $L$ is the Navier-Stokes operator, $\varphi$ is the velocity field and $p$ is the pressure. Using standard technique [1], one can construct a probability measure and, hence, a characteristic functional (CF) for the micromodel (1)

$$ Z[\eta, \tilde{\eta}] = \int e^{iS[\tilde{\varphi}, \varphi] + i\int \varphi \eta + i\int \tilde{\varphi} \tilde{\eta} \tilde{D} \tilde{\varphi} D \varphi}, \quad (2) $$

which is a field theory with action

$$ S[\tilde{\varphi}, \varphi] = -\int \tilde{\varphi}(1) L[\varphi, 1] d1 + \frac{i}{2} \int \tilde{\varphi}(1) D(12) \tilde{\varphi}(2) d1 d2, \quad (3) $$

where $D(12) = (f(1)f(2))$, $d1$ means $d\vec{x}_1 dt_1$. Since we regard an incompressible fluid, the pressure can be excluded from the action $S$, with both $\tilde{\varphi}, \varphi$ and random force considered as transversal:

$$ S(\Phi) = g_0 \nu_0^3 \frac{\tilde{\varphi} \tilde{D} \tilde{\varphi}}{2} + \varphi[-\partial_t \varphi - (\varphi \partial) \varphi + \nu_0 \Delta \varphi], \quad (4) $$

$$ D_{ij}(x, x') = D(x - x') P_{ij}, \quad (5) $$

where $g_0$ is the dimensional bare coupling constant, $D = D/g_0 \nu_0^3$, and

$$ P_{ij}(\vec{k}) = \delta_{ij} - \frac{k_i k_j}{k^2} $$

is an orthogonal projector. Hereafter, we denote $\Phi \equiv (\tilde{\varphi}, \varphi)$ and $A \equiv (\tilde{\eta}, \eta)$.

The field theory (4) is a renormalizable one, which needs only multiplicative renormalization of parameters, rather than fields

$$ \nu_0 = \nu Z_\nu, \quad g_0 = g M_2^2 Z_g, \quad Z_g = Z_\nu^{-3}, \quad (6) $$

where $Z_\nu, Z_g$ are the renormalization multipliers, $M$ is the scale setting, $Z_\Phi \equiv 1$, thus, no dynamical field renormalization is needed. The subscript "0" denotes the bare parameters, as usual. The dimensionless coupling constant $g$, the expansion parameter is introduced to make the theory of "real" quantum field theory form

$$ S_{\text{ren}} = g M_2^2 \nu^3 \frac{\tilde{\varphi} \tilde{D} \tilde{\varphi}}{2} + \tilde{\varphi}[-\partial_t \varphi - (\varphi \partial) \varphi + \nu \Delta \varphi]. \quad (7) $$

Up to this point nothing has been said about the random force correlation function $D(x', x)$. If the force $\delta$-correlated in time one (otherwise the process would be essentially non-Markovian), then $D$ should be taken in the form

$$ D(x', x) = \delta(t' - t) B(\vec{x}' - \vec{x}) = \text{const} \times \delta(t' - t)|\vec{x}' - \vec{x}|^{-4+2\varepsilon} \quad (8) $$