The method of moments of the Boltzmann-Vlasov equation is used to derive relations between the macroscopic quantities of high-temperature plasma (i.e. the density of particles, the mass velocity and the generally anisotropic tensor of the kinetic pressure of particles) and the static electromagnetic field by which the plasma is stationarily confined. The well-known relation between the isotropic pressure of particles and the pressure of the magnetic field is then obtained from this as a special case. Plane geometric configuration of the plasma boundary and negligible collision interaction of the particles are assumed. The results are verified on a simple kinetic model of plasma with known trajectories of the particles.

1. INTRODUCTION

When studying the instability of high-temperature plasma confined by the field of a magnetic trap a state of stationary equilibrium is assumed when all the parameters are time independent. Despite the fact that the effects caused by Coulomb collision interaction of the particles (e.g. the escape of particles through mirrors or other “openings” of the magnetic trap, the diffusion of plasma across the magnetic field) are thereby neglected, this approximation is justified since from the point of view of time development of instabilities (taking place practically with the thermal velocity of particles) the above processes are much slower.

In the majority of present-day magnetic traps, particularly those of the mirror type (e.g. Ogra or DCX), the plasma is very far from the state of thermodynamic equilibrium primarily because the confinement period is not sufficient for complete thermalization, not to mention the fact that the stationary confinement of a fully thermalized plasma is not possible [1], [2].

Such non-thermal plasma is characterized by the substantially anisotropic pressure. The usual equations of magnetohydrodynamics cannot therefore be used. The generalization of magnetohydrodynamics, the CGL approximation, so-called after its authors [3], is limited merely to the case of a very strong magnetic field and a special form of the kinetic pressure tensor is assumed. In the general case one must introduce the Boltzmann kinetic equation without the collision term (Boltzmann-Vlasov equation) or its equivalent — a system of moment equations [4], [5].

While in the case of stationary confinement of plasma with an isotropic pressure the continuity and Euler equations (together with the Maxwell equations — the so-called magnetohydrostatic equations) are sufficient for the calculation of the plasma parameters, in the general case of anisotropic pressure one must introduce in addition to the first two moment equations an equation for the energy transfer. If, then, we close the infinite chain of moment equations by a suitable
assumption as to the tensor of heat flow, we obtain a sufficient number of equations for determining all six components of the pressure tensor.

These equations are used in this paper to solve the case of an infinite plane layer of charged particles of one kind confined by homogeneous and mutually orthogonal magnetic and electric fields. It is seen that in this case the moment equations are solvable in the closed form in the sense that if the total electromagnetic field is given, the macroscopic quantities of the system of particles can be determined from the relations derived. The well-known relation between the isotropic pressure of the particles and the pressure of the magnetic field is obtained from this as a special case.

The results are then verified on a simple kinetic model of a plane layer, formed by mono-energetic particles describing cyclotron circles having the same radius and with centres placed in a common plane, while the collective electromagnetic field is much weaker than the external magnetic field. This model was chosen because it represents the extreme case of stationary configuration of charged particles which is formed in a mirror magnetic trap with transversal monoenergetic injection on the assumption that the trap is very long and the cyclotron radius much smaller than the distance of the injector from the axis of the trap.

2. MOMENT EQUATIONS OF STATIONARY STATE

The necessary first three moment equations of the stationary state of a system of charged particles of one kind are obtained from the well-known moment equations [4], [5] by putting the time derivative of all the quantities equal to zero. In the Cartesian coordinates $x_i$ the required equations have the following component form:

\begin{align*}
(1) & \quad \frac{\partial n u_i}{\partial x_i} = 0 \\
(2) & \quad m n u_j \frac{\partial u_i}{\partial x_j} + \frac{\partial P_{i j}}{\partial x_j} = e n E_i + \frac{1}{c} e u_i u_j H_j \\
(3) & \quad \frac{1}{2} \left( u_i \frac{\partial P_{j k}}{\partial x_i} + P_{j k} \frac{\partial u_i}{\partial x_i} + P_{i j} \frac{\partial u_k}{\partial x_i} + P_{i k} \frac{\partial u_j}{\partial x_i} \right) + \\
& \quad + \frac{\partial Q_{i j k}}{\partial x_i} - \frac{1}{2 m c} (e_{j r} H_r P_{i k} + e_{k r} H_r P_{i r}) = 0
\end{align*}

(1) is the continuity equation, (2) the Euler equation and (3) the equation for the energy transfer (equation of pressure flow); $n$ is the density of the particles, $u_i$ the vector of the mass velocity, $P_{i j}$ the tensor of the kinetic pressure, $Q_{i j k}$ the tensor of the heat flow, $E_i$ the electric field strength, $H_i$ the magnetic field strength, $e$ the charge and $m$ the mass of a particle, $c$ the velocity of light and $e_{i j k}$ the Levi-Civita anti-symmetric tensor.