DIFFUSION IN SOLID PHASE
WITH NONSTATIONARY INTERPHASE BOUNDARY

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This paper deals with the diffusion in a solid phase which is in contact with a melt. Due to dissolution and diffusion of the solid in the melt the interphase boundary moves. The mathematical description leads to the solution of the diffusion equation, the constant Dirichlet boundary condition of which is defined on the nonstationary boundary of the diffusion field. The problem is solved by means of the thermal potential of a double layer. The values of diffusion coefficients obtained from experimental data according to this theory are higher than in the case where the movement of the interphase boundary is not taken into account.

Introduction

The paper [1] deals with diffusion in the melt taking into account the movement of interphase boundary. There were considered, for example, metal A, which is immersed into molten metal B under the condition that the temperature of the melt B is lower than the liquidus temperature of metal A. In this paper we will follow the inverse case — the diffusion from the melt into the solid phase, that means the diffusion of metal B into metal A. With regard to the melting of the solid phase the boundary value of the diffusion equation will be given on a moving boundary. It deals with the Dirichlet boundary problem with a nonstationary boundary of the diffusion domain. This problem leads to the calculation of the thermal potential of a double layer and the solution of a Volterra integral equation of the second kind, from which the density of the thermal potential will be determined [2]. In view of the fact that the mathematical problems were described in paper [1], we shall limit ourselves in this paper to a brief statement of the process of solving, to its application in experimental practice and to the steps in partial numerical calculations.

Formulating the problem and defining the boundary conditions of the diffusion equation

Let us take into account the phase boundary at time \( t = 0 \) in point \( x = 0 \). On the interphase boundary between the solid metal A and the molten metal B a concentration jump exists. From the side of the melt phase there is the concentration

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c_{BL}, on the side of the solid phase the concentration $c_{BS}$, where $c_{BL}$ and $c_{BS}$ are the concentrations of liquidus and solidus according to the phase diagram — Fig. 1. The concentration jump magnitude is equal to $|c_{BL} - c_{BS}|$ — see [6]. As a result of the diffusion and because the value of the coefficient of diffusion $D_{AL}$ of the metal A into the melt B is considerably greater than the diffusion coefficient $D_{BS}$ of the metal B into the solid phase A, dissolution of the solid phase takes place. It appears that the interphase boundary moves into the domain $x > 0$. The motion of this boundary shall be described by means of the function $\chi(t)$.

Fig. 1. Scheme of diffusion in solid phase with nonstationary phase boundary. A — metal in a solid phase, B — molten metal, $c_{BS}$ — concentration of the solidus, $c_{BL}$ — concentration of the liquidus, $\chi(t)$ — moving interphase boundary.

Assuming that the diffusion coefficient $D_{BS}$ in the solid phase is independent of concentration, we will seek the solution of the diffusion equation

$$\frac{\partial c_B(x, t)}{\partial t} = D_{BS} \frac{\partial^2 c_B(x, t)}{\partial x^2} \quad (1)$$

which satisfies the boundary conditions

$$c_B(x, 0) = 0 \quad \text{for} \quad x \geq 0,$$
$$c_B[\chi(t), t] = c_{BS} \quad \text{for} \quad t > 0,$$  \quad (2)

where $c_B(x, t)$ is the spread of concentration of the element B in solid phase A. From papers [1, 4–6] it follows

$$\chi(t) = \alpha \sqrt{t}.$$  \quad (3)