Continuity Properties of Bargaining Solutions

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Abstract: Three different concepts of continuity of bargaining solutions are examined: Pareto continuity, Hausdorff continuity, and Kuratowski continuity. A new axiomatic characterization of the Nash solution is proposed. In this axiomatization, Kuratowski continuity plays a major role.

1 Introduction

In this paper three different concepts of continuity of bargaining solutions are studied: Kuratowski continuity, Hausdorff continuity, and Pareto continuity. Jansen and Tijs (1983) give a detailed analysis of Hausdorff continuity in the context of two-person bargaining games. The concept of Pareto continuity was introduced by Peters (1986a), and it has been utilized in several axiomatizations of bargaining solutions (see e.g. Chun and Thomson 1990a, and Peters 1986a). Kuratowski continuity is not as well known as the two other concepts. To my knowledge, Jansen and Tijs (1983) is the only paper, where this concept is explicitly used. They consider mostly games which have compact (and convex) feasible sets (feasible set is the set of all possible utility allocations for the players in the game). In this class, Hausdorff continuity is equivalent to Kuratowski continuity. But Jansen and Tijs also suggest that in the class of games with closed (not necessarily compact) feasible sets, Kuratowski continuity should be used. In this class, Kuratowski continuity implies Hausdorff continuity, but the concepts are no longer equivalent.

If we want to apply bargaining theory to "real" bargaining situations, solutions should exhibit some kind of continuity, since in practice it is impossible to know exactly the utility functions of the players. Intuitively, continuity means that small changes in the data that specifies the game should change the solution outcome only slightly. It depends on the continuity concept, what kind of changes in the data are considered as "small". Hausdorff continuity relies on Hausdorff topology: two games are close to each other, if their feasible sets are "close" to

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each other in the Hausdorff topology, and their disagreement points are close to each other. If the disagreement points are the same, and the Pareto sets are "close" to each other in the Hausdorff topology, then Pareto continuity demands that solutions must be close to each other.

Kuratowski continuity is a stronger concept than Hausdorff continuity. The way these concepts differ can be very important. Namely, it is often convenient to assume that the feasible sets are comprehensive. Comprehensiveness corresponds to the free disposal of utility assumption that is frequently used in microeconomics. On the other hand, compact feasible sets may seem more natural than comprehensive ones, since in "reality", choice sets are often bounded. In such cases we would like the solution of a comprehensive game to be quite close to a solution of some compact game. But in the Hausdorff topology, we can never approximate a comprehensive game by a sequence of compact games. Applying Kuratowski continuity (and the underlying convergence notion), such an approximating sequence can always be found. Therefore, if the solution is Kuratowski continuous, it doesn't matter too much, if we use a comprehensive or a (large) compact feasible set to represent the choices available to the players.

The paper is organized in the following way. In Section 2 the basic elements of axiomatic bargaining theory are introduced. The continuity concepts are defined formally in Section 3. Section 4 contains the results. It is first shown that while Kuratowski continuity implies Hausdorff continuity, neither of these concepts implies Pareto continuity in the class of games with compact (or closed) feasible sets. In the class of comprehensive games, Hausdorff continuity implies Pareto continuity, but there are Pareto continuous solutions which are not Hausdorff continuous, and Hausdorff continuous solutions which are not Kuratowski continuous. A new axiomatic characterization of the Nash solution is proposed. This axiomatization is closely related to the one given by Chun and Thomson (1990a). The only difference is that the axioms of Pareto continuity and independence of nonindividually rational alternatives are replaced by a single axiom, the Kuratowski continuity. These axiomatizations are compared to each other at the end of Section 4. Section 5 concludes.

2 Basic Concepts

An n-person bargaining game is a pair $(S, d)$, where $S \subset \mathbb{R}^n$ is a convex and closed set so that $d \in S$, and $d < x$ (i.e. $d_i < x_i$, $i \in \{1, \ldots, n\} = I$), for some $x \in S$, and $S$ contains at least one Pareto optimal point. The set of Pareto optimal points of a game $(S, d)$ is denoted by $P(S)$. $P(S) = \{x \in S | y \geq x, y \neq x \implies y \notin S\}$. Elements of $S$ are feasible utility allocations for players $1, 2, \ldots, n$. If they cannot reach an agreement, which element of $S$ should be chosen as the solution of the bargaining game $(S, d)$, the situation results in conflict, where player $i$ gets a utility level $d_i$. Let