Six Bits for Nine Colored Quarks

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Abstract

The hypercomplex number system of the Dirac equation is used to generalize $SU(2)$ to the covering group of $SO(4)$. The basic representations in this number language suggest a parton model of 6 “bits” and 6 “antibits”; one with spin 0, two with spin $\frac{1}{2}$, and three with spin 1. The relationship of this to the special relativity group is also considered.

Introduction

Though the original simple quark theory had to be expanded (three colored triplets), and recent high-energy experimental results (Ellis, 1974) cast more doubt on the popular parton models, the parton concept may yet survive in some modified form. The $SU(2)$ description of isospin is certainly a useful concept. The empirical concept of strangeness introduced a generalization of the group. Thus, $SU(3)$ seems a rather “natural” extension, when $SU(2)$ is written in the standard $2 \times 2$ complex matrix form. There are well known difficulties with $SU(3)$ in relation to Lorentz covariance. This is very serious, since the nuclear force is strong and one should expect that a relativistic treatment is required to describe the partons that make up a proton, neutron, etc.

In this paper we would like to show that $SU(2)$ can be generalized in a ‘natural’ way, leading to the possibility of 6 basic partons and 6 antipartons, by casting it in hypercomplex number form. This generalization is compatible with special relativity if generalized to rotations in $(4, 1)$ space-time. We have argued previously that this generalization is reasonable and connected with the existence of rest mass in nature. Here we shall only show how the group structure is formulated and how it suggests the number and spins of the partons. The quark model, though suggested by $SU(3)$, has much of its success without reference to the original motivating group. Our development may contain a similar pattern.

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1. \( SU(2) \) in Quaternion Form

The Pauli matrices \( \sigma_k \) along with \( \sigma_0 = 1 \) form a basis for the complex quaternion number ring when complex coefficients are used. The hermitian conjugate \( A^* \) is supplemented by the quaternion conjugate \( A^\dagger \), where \( \sigma_0^\dagger \equiv \sigma_0, \sigma_k^\dagger \equiv -\sigma_k \), and \( C^\dagger \equiv C \) for \( C \) a complex coefficient. We then have

\[
A = A^\mu \sigma_\mu + i A^I \sigma_I \quad \text{and} \quad (AB)^* = B^* A^*, \quad (AB)^\dagger = B^\dagger A^\dagger
\]  

(1.1)

The "spinor representation" of the rotation group now takes the form

\[
\psi_a' \equiv R^\dagger \psi_a, \quad \psi_v' \equiv R^* \psi_v, \quad RR^\dagger \equiv 1 \sigma_0, \quad R^\dagger \equiv R^*, \quad \Rightarrow \psi_a = \psi_v \equiv \psi
\]  

(1.2)

An infinitesimal rotation satisfies

\[
R = 1 + \delta \sigma_k, \quad \delta \delta \approx 0, \Rightarrow \delta^\dagger = -\delta = \delta^* \Rightarrow \text{generators } i \sigma_k
\]  

(1.3)

Since \( R \) has a \( 2 \times 2 \) complex matrix representation, \( \psi \) has a \( 1 \times 2 \) representation, and therefore two complex number parts (four real parameters). These parts are associated with \( \psi \) (proton) and \( \psi \) (neutron) in isospin.

For completeness, we remark that \( LL^\dagger \approx 1 \sigma_0 \) gives the Lorentz group, with generators \( i \sigma_k \) and \( \sigma_k \) since \( L^* \neq L^\dagger \) in general. The Lorentz (space-time) transformation is given by \( x' = x^\mu \sigma_\mu = L^* x L \), and \( (x|x) = x^\mu x^\nu \sigma_\mu \sigma_\nu \).

2. Generalized Number System

Because of rest mass, the Weyl equation, \( i \hbar \partial^\mu \sigma_\mu \psi_a = 0 \), is replaced by the Dirac equation, \( i \hbar \partial^\mu (e_\mu) \psi_a = mc(ifo) \psi_a \), and the complex quaternion number system \( \{e_\mu, i\sigma_\mu\} \) is generalized to a direct product number system with basis elements \( \{\sigma_0, i\sigma_1, \sigma_2, \sigma_3\} \otimes \{\sigma_0, i\sigma_1, i\sigma_2, i\sigma_3\} \); a 16-element number system with real coefficients. The basis can have the following matrix representation

\[
(e_\mu) \equiv \begin{pmatrix}
\sigma_\mu & 0 \\
0 & \sigma_\mu^\dagger
\end{pmatrix}, \quad (i\sigma_\mu) \equiv \begin{pmatrix}
(i\sigma_\mu) & 0 \\
0 & (i\sigma_\mu)^\dagger
\end{pmatrix}, \quad (f_\mu) \equiv \begin{pmatrix}
0 & \sigma_\mu^\dagger \\
\sigma_\mu & 0
\end{pmatrix},
\]

\[
(i\sigma_\mu) \equiv \begin{pmatrix}
0 & (i\sigma_\mu)^\dagger \\
(i\sigma_\mu) & 0
\end{pmatrix}
\]  

(2.1)

The hermitian conjugate, now written \( (\ )^\dagger \), gives

\[
(e_\mu)^\dagger = (e_\mu), \quad (i\sigma_\mu)^\dagger = -(i\sigma_\mu), \quad (f_\mu)^\dagger = (f_\mu), \quad (f_k)^\dagger = -(f_k),
\]

\[
(i\sigma_0)^\dagger = (i\sigma_0), \quad (i\sigma_k)^\dagger = -(i\sigma_k)
\]  

(2.2)

We generalize the quaternion conjugate \( (\ )^\dagger \) to \( (\ )^\sim \) as follows

\[
(e_0)^\sim \equiv (e_0), \quad (e_k)^\sim \equiv -(e_k), \quad (i\sigma_0)^\sim \equiv (i\sigma_0), \quad (i\sigma_k)^\sim \equiv -(i\sigma_k),
\]

\[
(f_\mu)^\sim \equiv (f_\mu), \quad (i\sigma_\mu)^\sim \equiv -(i\sigma_\mu)
\]  

(2.3)