Group Analysis of Masses and Spins in Curved Space-Time: Cosmological and Experimental Consequences

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Abstract

Recent developments in spontaneously broken gauge theories as well as in group analysis of masses and spins in curved space-time indicate that rest masses may change as a function of cosmic time. Such an effect is incompatible with standard cosmological models. A set of cosmological models that incorporate mass variation is introduced. These cosmological models are shown to be fully compatible with the group analysis, yielding exactly the same formula; they are used therefore as a theoretical testing ground for the hypothesis of mass variation. The following consequences of this hypothesis are obtained: (1) Cosmological red-shifts are shown to correspond to a contracting, rather than expanding, universe. (2) The effects of mass variation on planetary orbits are calculated; they are not precluded by the data. Conclusive experimental evidence is expected within a few years.

1. Introduction

For space-time with constant four-dimensional curvature the theory of group representations reveals a deep relationship between the geometry of the universe and fundamental properties of particles: Eigenvalues of the Casimir operators of the group of motion correspond to masses and spins (Wigner, 1939; Thomas, 1941). Realistic cosmological models, however, have a four-dimensional curvature $K(t)$ which is a function of cosmic time $t$. Such models do not have a group of motion. A relationship between the geometry of the model and fundamental properties of particles was nevertheless derived (Malin,
1974), and it led to the following result: While spins of particles are rigorously conserved, all masses change as a function of cosmic time,\(^3\) decrease in an expanding universe, and increase in a contracting one.

For the standard cosmological models of general relativity, rest masses are conserved. One needs, therefore, a different set of cosmological models to serve as a theoretical testing ground for the hypothesis of mass variation. Such a set can be obtained from the equations

\[
R_{\mu\nu} = -4\pi\kappa T_{\mu\nu}
\]  

(Malin, 1975).\(^4\)\(^5\) Equations (1.1) cannot, in general, replace Einstein’s equations, because their hydrodynamic consequences are inconsistent with experimental results (Lindblom & Nester, 1975). The cosmological models that are derived from equation (1.1) for isotropic, spatially homogeneous universes, however, are not precluded by observational and experimental data and incorporate time variation of all masses. They can serve therefore as a theoretical testing ground for the hypothesis of mass variation.

In the present paper these cosmological models and the group analysis of masses and spins in curved space-time are shown to yield exactly the same result for mass variation. We then proceed to derive consequences of such a mass variation for the interpretation of the cosmological red-shift and for planetary orbits.

Section 2 presents the cosmological models of equation (1.1). Previous results are summarized and some new relationships are obtained. All the models are shown (a) to have positive four-dimensional curvature \(K\) (as compared with the standard cosmological models of general relativity, all having \(K < 0\)); (b) to imply a time variation of all masses \(m\) according to the formula

\[
m(t) \sim \left[ K(t) \right]^{1/2}
\]

(1.2)

In Sections 3–5 the method of group analysis of masses and spins (Malin, 1974) is applied to cosmological models with \(K > 0\). The relevant group is \(O(3, 2)\), as compared with \(O(4, 1)\) for the case \(K < 0\). As pointed out by Dirac (1935) unresolved difficulties in formulating quantum theory in space-times

\(^3\) Recently the possibility of mass variation was also raised in the context of Salam’s (1968) and Weinberg’s (1967) spontaneously broken gauge theories of weak and electromagnetic interactions. Spontaneously broken gauge theories were reviewed by Mahanthappa (1973) and Abers & Lee (1973). Consequences of such theories for the gravitational field equations were discussed by Linde (1974), Dreitlein (1974) and Veltman (1974). In the context of such theories the energy of the vacuum depends on the temperature of the medium and therefore varies on the cosmological scale (Kirzhnitz, 1972; Kirzhnitz & Linde, 1972; Weinberg, 1974). Various rest masses may vanish to zeroth order and can then be calculated as higher-order effects (Weinberg, 1972a; Georgi & Glashow, 1972). Such effects may change with the temperature of the medium and therefore change with time.

\(^4\) \(R_{\mu\nu}\) is the Ricci tensor, \(R\) is the Riemann scalar, \(\kappa\) is the gravitational constant, and \(T_{\mu\nu}\) is the energy-momentum tensor.

\(^5\) The constant \(4\pi\kappa\) in equation (1.1) was derived by the requirement that Newton’s law of gravitation is obtained as a limiting case, in complete analogy with the derivation of the constant \(8\pi\kappa\) in Einstein’s equations.