Improvement in the frequency response of the electrochemical wall shear stress meter*

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Abstract. One of the very few disadvantages of the mass-transfer transducer when compared with the hot-film sensor, is a slightly diminished frequency-response due to the higher Prandtl number encountered. Mass or thermal balance and transfer equations were solved first by Fortuna and Hanratty (1971) for small fluctuations of the wall shear. The solutions allow to make accurate corrections on the frequency spectra and the power of the fluctuations, but in different time. In this paper, the author deduces the frequency response of split rectangular electrodes and shows how a combination of signals improves the response at higher frequencies and makes it comparable to the thermal transducer with the same size, in the same fluid. Two experimental devices are described and compared. With these devices, the measurement of the wall shear fluctuations is improved in real time. Accurate determinations of turbulent power fluctuations and probability density spectra are feasible and illustrate the subject.

List of symbols

- $A$: total area of the electrode
- $A_j$: area of the part $j$ of the electrode
- $a$: coefficient
- $C$: concentration
- $C_o$: bulk concentration
- $c$: fluctuation of concentration
- $D$: diffusion coefficient
- $F$: Faraday’s constant
- $f(n^*)$: transfer function
- $g$: gain of the differential electrode
- $i$: electrolysis current on the part $j$ of the electrode
- $K$: transfer coefficient
- $k$: fluctuation of $K$
- $l$: electrode length
- $n$: frequency
- $P_r$: Prandtl number
- $S$: wall shear
- $s$: fluctuation of the wall-shear
- $t$: time
- $x$: direction of the flow
- $y$: direction normal to the wall
- $\Delta \theta$: phase delay
- $\nu$: kinematic viscosity

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1 Introduction

The transfer to a surface electrode, in the conditions of use of the wall-shear meter, is governed by two equations. The mass balance which is formally analogous to the thermal balance for a hot-film, is written for a flow in the $x$ direction, by:

$$\frac{\partial C}{\partial t} + S \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2}$$

$S$ is the velocity gradient at the wall and $D$ the diffusion coefficient. The concentration $C$ is zero on the electrode. This equation describes the mass boundary layer which develops over a split electrode and in its wake (Fig. 1).

The transfer equation connects the transfer coefficient $K_j$ on any part $j$ of the electrode, to the surface concentration gradient:

$$K_j = \frac{D}{C_o} \frac{1}{A_j} \int_0^y \frac{\partial C}{\partial y} \, dA$$

$C_o$ is the bulk concentration, $A_j$ the area of the part $j$ of the electrode and $A$ its total area.

Fig. 1. Universal two-dimensional concentration field over an electrode
For the electrochemical meter, the transfer coefficient is directly proportional to the intensity \( I_j \) of the electrolysis current measured in experiments:

\[
K_j = \frac{1}{C_x A dF} \frac{I_j}{dF},
\]

so that we can refer to \( K_j \) as the signal on the part \( j \) of the electrode.

The following transformation in which \( l \) is the total electrode length, \( t \) the time and \( n \) the frequency:

\[
C^+ = \frac{C}{C_x} \quad x^+ = \frac{x}{l} \quad y^+ = \frac{y}{S^{+1/3}}
\]

renders dimensionless the mass balance and the transfer equation:

\[
\frac{\partial C^+}{\partial t^+} + y^+ \frac{\partial C^+}{\partial x^+} = \frac{\partial^2 C^+}{\partial y^+^2}
\]

\[
K_j^+ = \frac{S^+}{D^+} \frac{K_j}{D} \quad n^+ = \frac{n l^2}{D S^{+2/3}}
\]

Neglecting the second order terms, the mass balance (5) can be written as:

\[
\frac{\partial C^+}{\partial t^+} + y^+ \frac{\partial C^+}{\partial x^+} + \frac{s}{S^+} y^+ \frac{\partial C^+}{\partial y^+} + y^+ \frac{\partial C^+}{\partial x^+} = \frac{\partial^2 C^+}{\partial y^+^2} + \frac{\partial^2 c^+}{\partial y^+^2}
\]

or, subtracting the time-averaged values:

\[
\frac{\partial c^+}{\partial t^+} + \frac{s}{S^+} y^+ \frac{\partial C^+}{\partial x^+} + y^+ \frac{\partial c^+}{\partial x^+} = \frac{\partial^2 c^+}{\partial y^+^2}.
\]

Mitchell and Hanratty (1966) showed that neglecting nonlinear second-order terms in the mass balance introduces an error less than 3% for relative velocity intensities of 0.5. For a sinusoidal fluctuation of the wall shear:

\[
\frac{s}{S^+} = \frac{2\pi n^+ \epsilon}{S} \quad \text{and} \quad c^+ = \frac{\epsilon}{S} \frac{2\pi n^+ \epsilon}{S}
\]

and the mass balance only depends on one parameter, the dimensionless frequency \( n^+ \):

\[
i 2\pi n^+ \epsilon + y^+ \frac{\partial c^+}{\partial x^+} + y^+ \frac{\partial c^+}{\partial x^+} = \frac{\partial^2 c^+}{\partial y^+^2}.
\]

The transfer Eq. (6) allows to define the transfer function \( f(n^+) \) of the complete electrode by:

\[
\frac{1}{f(n^+)} = \frac{1}{\frac{1}{S}\int_A \frac{\partial c^+}{\partial y^+} \bigg|_{y^+ = 0} dA} = \frac{1}{\frac{1}{S}\int_A \frac{\partial c^+}{\partial y^+} \bigg|_{y^+ = 0} dA} = \frac{1}{8} f(n^+) = f(n^+) e^{i\theta}
\]

Numerical solutions of (13) and (14) were executed first by Fortuna and Hanratty (1971) as a function of two parameters. The results, related to the absolute value of the transfer function \( |f(n^+)| \), were grouped together as a function of the dimensionless frequency \( n^+ \) (Hanratty and Chorn 1975). Py (1973) represented \( f(n^+) \) in modulus and phase displacement \( \Delta \theta \) for different sorts of electrodes. A good analytical representation of \( f(n^+) \) up to \( n^+ = 5 \) is given by the formulae:

\[
|f(n^+)| = \frac{1}{\sqrt{1 + 2.369 n^{+2.5}}}
\]

\[
\Delta \theta = 1.954 n^{+1.4} \quad 0.4 + n^{+1.4}
\]

\[
f(n^+) = |f(n^+)| e^{i\Delta \theta}
\]

\([f(n^+)]\) represents the damping of the signal and \( \Delta \theta \) the phase delay in radians. These quantities are represented in Fig. 2, curves A. A progressive damping occurs from \( n^+ = 0.2 \) till higher frequencies. The phase delay appears at \( n^+ = 0.03 \).

This was more recently recalculated by Mao and Hanratty (1985) and Ambari et al. (1986). The formers showed how the electrode lengths can be smaller with the mass probes which improves the frequency response [Eq. (4)].