EFFECTIVENESS OF A SYSTEM OF ROD ABSORBERS
IN A REACTOR FITTED WITH A REFLECTOR

V. I. Nosov

Translated from Atomnaya Énergiya, Vol. 9, No. 4, pp. 262-269, October, 1960
Original article submitted November 19, 1959

The conditions of criticality and the distributions of the neutron flux for a homogeneous thermal-neutron reactor with a system of absorbing rods are obtained in the two-group approximation. The rods extend through the entire depth of the reactor and are situated around the circumference of the active zone or radial reflector at a uniform distance from one another. The results of the calculation are presented.

Introduction

In a number of articles in the literature [1-3], the effectiveness of a system of rods located in the active zone of thermal reactors is calculated in a simplified way for the case of reactors without reflectors. However, in some cases such an approximation is not sufficient, and it is required to calculate the effectiveness of the system of rods in the active zone of a reactor with a reflector, where the control rods can be located in the reflector itself. This article is devoted to the consideration of the effectiveness of a system of rods in the two-group approximation.

Formulation of the Problem

We shall write the two-group equations for the moderator density \( q(r) \) and the thermal-neutron density \( n(r) \) [4]:

\[
\begin{align*}
\tau \Delta q(r) - q(r) + \frac{k_{\infty} n(r)}{k_{\text{eff}}} &= 0; \\
L^2 \Delta n(r) - n(r) + p l q(r) &= 0,
\end{align*}
\]

where \( \tau \) is the neutron age; \( L^2 \) is the square of the thermal-neutron diffusion length; \( l \) is the lifetime of thermal neutrons in an infinite medium; \( p \) is the probability of avoiding resonance capture; \( k_{\infty} \) is the neutron multiplication factor in an infinite medium; \( k_{\text{eff}} \) is the effective multiplication factor; \( q(r) \) and \( n(r) \) are functions of the coordinates \( r, \varphi, \) and \( z. \)

The solution for \( q \) and \( n \) in a cylindrical reactor without end reflectors has the form

\[
q = S_1 \psi_1 + S_2 \psi_2; \quad n = \psi_1 + \psi_2.
\]

Here \( S_1 \) and \( S_2 \) are the coupling coefficients:

\[
\psi_1 = \sum_{n=0}^{\infty} \left[ A_{1n} J_n \left( \sqrt{x_1^2 - (\pi/H_{\text{ex}})^2} \right) + B_{1n} Y_n \left( \sqrt{x_1^2 - (\pi/H_{\text{ex}})^2} \right) \right] \cos n \varphi + E_{1n} \sin n \varphi;
\]

\( 795 \)
where $A_n$, $B_n$, and $E_q$ are constants determined from the boundary conditions; $J_n$, $Y_n$, $I_n$, and $K_n$ are Bessel functions [5]; $\kappa_1^2$, $\kappa_2^2$ are the roots of the two-group equation of criticality

$\frac{\alpha_{\infty}}{k_{\text{eff}}} = (1 + \alpha^2 r) (1 + \alpha^4 L^2)$;

$H_{\text{ex}}$ is the reactor height and includes the extrapolation distance and effective additions from the end reflectors. In obtaining relations (2a) and (2b), we assumed that it is possible to separate the variables and that the first root of Eq (2c) is positive, since in the reactors of interest to us one usually has $\frac{\alpha_{\infty}}{k_{\text{eff}}} > 1$.

The solution for the neutron flux with a system of rods in the reactor can be represented as the superposition of two partial solutions, one of which is always regular, while the other (irregular) has singularities at the absorbing rods:

$q = S_1 (\psi_1^{(R)} + \psi_1^{(ir)}) + S_2 (\psi_2^{(R)} + \psi_2^{(ir)}); n = (\psi_1^{(R)} + \psi_1^{(ir)}) + (\psi_2^{(R)} + \psi_2^{(ir)})$, (3)

where the superscript $R$ refers to the regular solution and $\text{ir}$ to the irregular solution.

The solutions (3) for $q$ and $n$ should satisfy the following boundary conditions:

1) $q$ and $n$ are bounded at each point of the reactor and vanish at the extrapolated radius of the reactor $R_{\text{ex}}$.

2) At the boundary between the active zone and the reflector the following relations hold:

$n^I = n^{II}; \quad \frac{dn^I}{dr} = \gamma_0 \frac{dn^{II}}{dr}; \quad q^I = \gamma_1 q^{II}; \quad \frac{dq^I}{dr} = \gamma_2 \frac{dq^{II}}{dr}$, (4)

where I and II refer to the active zone and reflector, respectively.

3) At the surface of the rod, the conditions for $q$ and $n$ have the form [6, 7]

$\frac{dq/dq}{q} \bigg|_{q=a} = \frac{1}{\alpha} \bigg|_{q=a}; \quad \frac{dn/dq}{n} \bigg|_{q=a} = \gamma \bigg|_{q=a}$, (5)

where $a$ is the geometrical radius of the rod.

We shall now proceed directly to the derivation of the equations of criticality.

System of Rods in the Reactor Reflector

In a reactor with a system of rods located at the circumference of the reflector at a uniform distance from one another (Fig. 1), the solution for the thermal neutron density and moderator density has the form

*If the addition theorem is used, the terms with $I_m(\nu \rho t)$ and $I_m(\mu \rho t)$ can be combined with the terms containing $I_nN(\nu t)$ and $I_nN(\mu t)$, respectively.

796