The resonance interaction of the radial and vertical oscillations close to \( n = 0.25 \) is considered. It has been shown that this resonance is considerably more harmful than the parametric excitation of the vertical oscillation, which is caused by the first harmonic in the magnetic field structure.

In actual high-energy phasotrons (400-700 Mev), the limiting energy to which the particles are accelerated corresponds to the radius at which the exponent of the magnetic-field fall-off lies in the limits \( 0.25 > n > 0.2 \). The coupled oscillations in the zone \( n = 0.2 \) cannot lead to the complete loss of the beam [1]. The region of parametric excitation of vertical oscillations at a frequency \( Q_z = 0.5 \) (\( n = 0.25 \)) penetrates directly into this zone. In actual phasotrons, parametric excitation cannot cause an essential increase in the amplitude, since the width of the resonance zone for a first harmonic of \( \epsilon_1 \approx 0.001 \) is usually less than 100 revolutions [2].

We shall consider the effects which may be evoked by an increase in the vertical oscillation amplitude in the presence of an azimuthal inhomogeneity of the magnetic field.

In this case, the betatron oscillations \((r, z, \varphi)\) are described by the equations *

\[
\begin{align*}
\dot{z}^* + [n + \epsilon_1 \cos (\varphi + \varphi_0)] z - \frac{2}{r} (d - n) \dot{z} + \frac{z'^2}{R} &= 0, \\
\dot{\varphi} + (1 - n) \dot{q} + \frac{1}{2} \frac{d}{R} \frac{\partial^2 H}{\partial z^2} - \frac{\epsilon_1^2}{2R} \frac{1}{2} (2d - n) z^2 + \frac{4}{2R} z'^2 &= -\epsilon_1 R \cos (\varphi + \varphi_0),
\end{align*}
\]

where \( n = -\frac{R}{H(R)} \frac{dH}{dr} \bigg|_{r=R} \); \( d = \frac{1}{2} \frac{R^2}{H(R)} \frac{d^2 H}{dr^2} \bigg|_{r=R} \); \( q = r - R \); \( H_z = H(r) [1 + \epsilon_1 \cos (\varphi + \varphi_0)] \) (the prime denotes differentiation with respect to the azimuthal angle).

We shall consider the coupled oscillations which result from the distortion of the closed orbit of the azimuthal magnetic field with an inhomogeneity in the structure. The equation for the vertical oscillations in this case can be represented in the form

\[
z^* + \left\{ n + \frac{\epsilon_1^2 d^2}{n^2(1-n)} + \epsilon_1 n \left[ 1 + \frac{2(d-n)}{n^2} \right] \cos (\varphi + \varphi_0) \right\} z = 0.
\]

* In the system (1), only the resonance terms containing \( \epsilon_1 \) are retained.
For all actual phasotrons, the quantity \( \frac{2|d - n|}{n^2} \gg 1 \) for \( n = 0.25 \), i.e., the beam loss at the limiting radii due to the coupling of the oscillations and not from the parametric excitation of the vertical oscillations of the first magnetic field harmonic. From Eq. (2), it follows that the oscillation amplitude is 2.7 times as great for \( v \) revolutions, where

\[
\nu = \frac{n}{2\pi e_1 (d - n)}.
\]

Taking into account the fact that \( d = -\frac{r}{2} \frac{dn}{dr} \), we readily obtain the radial width of the resonance zone:

\[
\Delta r = \frac{e_1}{n} R.
\]

If the average increase in energy in the accelerator is equal to \( eV \), then the number of revolutions for the width of the resonance zone is

\[
\nu = \frac{1 - \eta}{n} \frac{E_k}{eV} \frac{2E_o + E_k}{E_o + E_k},
\]

where \( E_k \) is the kinetic energy of the field. Thus, under conditions of quasi-static operation, one can estimate from (3) and (5) the increase in the amplitude of an ion in the resonance zone:

\[
\ln \frac{a_{\text{max}}}{a_0} = 2\pi e_1 d \frac{1 - \eta}{n} \frac{E_k}{eV} \frac{2E_o + E_k}{E_o + E_k}.
\]

Using the method of averaging [3], we can estimate the increase in the amplitude for dynamical operation:

\[
\ln \frac{a_{\text{max}}}{a_0} = 4V \frac{e_1 d}{V \chi},
\]

where \( \chi \) is the rate of change of the characteristic frequency in the resonance zone \( n = 0.25 \). This rate is determined from the expression

\[
\chi = \frac{4d t}{3\pi E_k} \frac{E_o + E_k}{2E_o + E_k}.
\]

The results were checked on an EMU-8 electronic simulator. The simulator made it possible to solve the problem of the dynamic passage of particles through the resonance of the coupling with forced radial oscillations for \( n = 0.25 \). Equation (2), which could be represented in the system of two equations

\[
z'' + n (\varphi) z' + \frac{2n_0 d}{n} Uz = 0,
\]

\[
U'' + U = 0,
\]

where \( n (\varphi) = n_0 + \chi \varphi; U (\varphi_0) = 1; U' (\varphi_0) = 0; z (\varphi_0) = n_0; z' (\varphi_0) = 0 \) was integrated on the EMU-8.

Since the time was used as the variable in the solution of the system (9) on the electronic simulator, it was possible to make an arbitrary choice of the initial phase \( \varphi_0 \).

In the integration, \( \varphi_0 \) was chosen in such a way so as to obtain the maximum increase in amplitude of the vertical oscillations in the process of the particle passing through the resonance of the coupling for \( n_0 = 0.25 \).

The maximum oscillation amplitude was observed directly on the screen of the indicator and was determined from the voltage at the output of the integrator. The accuracy of the solution is 1%.