One investigates the question of the exact bounds of the distribution of an arbitrary Gaussian linear measurable functional with respect to some conditional Gaussian measure, generated by a convex subset of a space with a Gaussian measure.

Let \((E, \gamma)\) be a linear space with a Gaussian measure [5], which is a Lebesgue space (see [3]). Let \(H \subseteq L^2(E, \gamma)\) be a Gaussian subspace, i.e., a closed linear subspace consisting of linear measurable functions with centralized Gaussian distributions. The space \(H\) is provided with a Hilbert norm, induced by the imbedding \(H \subseteq L^2(E, \gamma)\). The linear space \(E\) (considered within the accuracy of a completion to a linear measurable subset of complete measure) contains necessarily the subspace \(H^* \subseteq E\) (kernel) consisting of all continuous linear functionals on the Hilbert space \(H\) of Gaussian variables. The kernel \(H^*\) coincides with the set of quasiinvariant shifts of the space \((E, \gamma)\) and does not depend on the concrete realization of this linear space with a measure.

In this note we investigate the function \(J_K(x)\), the oscillation in the sense of Itô and Nisio (see [1, 2]), defined on a convex subset \(K\) of the space \(H\). M. A. Lifshits has pointed out the interesting phenomenon, inherent to infinite-dimensional Gaussian distributions: there exist convex cones of positive measure in the space \(E\) with a Gaussian measure \(\gamma\) and Gaussian variables \(x \in H\), almost surely nonnegative on the cone, such that with the aid of an "affine hyperplane" in \(E\), defined by the functional \(x\), one can cut off a nontrivial part of the cone, having zero measure. In other words, the distribution of the functional \(x\) with respect to the corresponding conditional Gaussian measure on the cone is concentrated on the ray \([a, +\infty)\) with \(a > 0\). We show that in terms of the behavior of the oscillation \(J_K\) on an arbitrary convex GB-set \(K \subseteq H\), for the given Gaussian variable \(x \in H\) one can indicate the exact bounds of the distribution of this variable with respect to the corresponding conditional Gaussian measure. As a consequence, for the Lifshits cones one calculates the exact value of the number \(a\).

Let \(K\) be some subset of Gaussian variables from the space \(H\). For the process \(K\) one has the following theorems of K. Itô and M. Nisio on nonrandom oscillations (see [1, 2]).

**Theorem 1.** There exists a function \(J_K : K \rightarrow [0, \infty]\) such that with probability 1 for all \(x \in K\) one has simultaneously the equality

\[
\lim_{y \rightarrow x, y \in K} \sup \left\{ y \right\} - \lim_{y \rightarrow x, y \in K} \inf \left\{ y \right\} = J_K(x).
\]
THEOREM 2. For each $x \in K$ individually, with probability 1 we have the equalities
\[
\lim \sup_{y \to x, \ y \in K} y = x + \frac{1}{2} \lambda_K(x),
\]
\[
\lim \inf_{y \to x, \ y \in K} y = x - \frac{1}{2} \lambda_K(x).
\]

Let $K$ be a convex subset of the space $H$ and let $\alpha(x)$ be the value of the oscillation in the sense of Itô and Nisio at the point $x \in K$.

Proposition. a) The oscillation $\alpha(x)$ is a concave function of $x \in K$, i.e., $\forall x, y \in K$ we have
\[
\lambda(\lambda x + (1-\lambda)y) \leq \lambda \lambda(x) + (1-\lambda) \lambda(y).
\]
b) The oscillation $\alpha$ is continuous on each segment lying in $K$.
c) If $0 \in K$, then $\inf_{z \in K} \lambda(z) = (1-\lambda) \lambda(0)$ for any $\lambda \in [0,1]$.

Proof. a) Let $x, y \in K$ and $\lambda \in [0,1]$. We select in $K$ sequences $\{x_n, n \geq 1\}$ and $\{y_n, n \geq 1\}$ such that with probability 1 one has the relations
\[
\lim \sup_{x_n \to x} x_n - \lim \inf_{x_n \to x} x_n = \lambda(x),
\]
\[
\lim \sup_{y_n \to y} y_n - \lim \inf_{y_n \to y} y_n = \lambda(y).
\]
Then we obtain
\[
\lambda(\lambda x + (1-\lambda)y) \geq \lim \sup_{x_n \to x} (\lambda x_n + (1-\lambda)y_n) - \lim \inf_{y_n \to y} (\lambda x_n + (1-\lambda)y_n) \geq
\]
\[
= \lim \sup_{x_n \to x} \lambda x_n + \lim \sup_{y_n \to y} (1-\lambda)y_n - \lim \inf_{x_n \to x} (1-\lambda)y_n - \lambda \lambda(x) + (1-\lambda) \lambda(y).
\]

The concavity of the oscillation $\alpha$ is proved.

b) Assume that the segment $[x, y]$ lies in $K$ and $\lambda(z) < \infty$ for $[x, y]$. By virtue of the concavity of $\alpha$ we obtain that it is continuous at all points of the segment with the possible exception of the endpoints. Directly from the definition of the oscillation in the sense of Itô and Nisio, we have that $\lim \sup_{z \to x, z \in K} \lambda(z) < \lambda(x)$. On the other hand, by virtue of the concavity of the oscillation for each $z \in [x, y]$ sufficiently close to $x$, we shall have the inequality $\lambda(x) < \lambda(z)$, i.e., $\lambda(z) \to \lambda(x)$ for $z \to x, z \in [x, y]$. Thus, the continuity of the oscillation $\alpha$ on the entire segment $[x, y]$ is proved.

c) We assume that $\inf_{z \in K} \lambda(z) = \lambda_0 \lambda(0)$ for some $\lambda_0 \in [0,1]$. We select $\varepsilon > 0$ such that $4 \varepsilon < \lambda - (1-\lambda) \lambda(0)$. We set $D_\varepsilon(x) = \{z \in K: |z-x| < \varepsilon\}$. We fix an arbitrary positive $\delta < \frac{1}{2}$. From the second theorem of Itô and Nisio it follows that there exists $\tau > 0$ such that the event $A_\delta = \{\sup_{z \in D_\varepsilon(0)} |z-z| < \frac{1}{2}\}$ occurs with a probability greater than $1 - \delta$. We select a sequence $\{x_n\} \subset K$ such that $\lim \sup_{x_n \to 0} x_n = \frac{1}{2} \lambda(0)$ a.s. We note that $\lim \sup_{x_n \to 0} \lambda x_n = \frac{1}{2} \lambda(0)$ for any $\lambda \in [0,1]$. We select a sequence of positive numbers $\{\delta_n, n \geq 1\}$ such that $\sum_{n=1}^{\infty} \delta_n - \delta$. There exist $\tau_n > 0, n \geq 1$, such that $D_{\tau_n}(\lambda_0 x_n) \subset D_{\tau}(0), n \geq 1$ and the events