Determination of the field and the forces on a current filament moving above a conducting plate

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Contents: It is well known that inside a conducting medium eddy currents are induced when an excitation of a time varying current source is placed in the vicinity of this medium. It is also of great interest the examination of the problem related to the motion of an exciting source and the induced eddy currents within a conducting material placed close to the moving source. The determination of the eddy current density and the field outside the conducting material due to the eddy currents gives the possibility of calculating the lift and drag force acting on the moving system.

Bestimmung der Feldgrößen und der Kräfte auf einem Stromfaden der sich über eine leitende Platte bewegt

Übersicht: Es ist wohl bekannt, daß innerhalb von einem leitenden Medium Wirbelströme induziert werden, wenn in der Nähe dieses Mediums eine zeitlich veränderliche Stromerregung vorhanden ist. Die Probleme also, die die Bewegung einer Erregungsquelle und die dadurch induzierten Wirbelströme innerhalb eines leitenden Mediums in der Nähe der Erregung betreffen, sind von großem Interesse. Die Bestimmung der Wirbelstromdichte und des Feldes außerhalb des leitenden Materials infolge der Wirbelströme ermöglicht die Bestimmung der Trag- und Bremskräfte, die auf das sich bewegende Teil wirken.

1 Introduction

In a previous work [1] it was examined the general case of a moving current loop and the case of two moving current filaments as an application. The problem of a vertical circular current loop moving above a conducting slab was treated in another work [2]. In the first paper the analysis gives a general expression for the eddy current density within the slab, which is considered having a very thin thickness. An important result of the paper is that the current density inside the conducting slab does not have a z-component. In the second paper the above result is verified and the current density is described by the x and y components.

In the present work an attempt is made for the calculation of the current density of the induced eddy current within a conducting slab of finite thickness, when the excitation is two current filaments moving above the slab with a constant velocity \( v \). The method which is followed is based on a double Fourier transformation with respect to time and one of the space variables. The excitation current is expressed as two-\( \delta \)-functions moving with constant velocity. The known Fast Fourier algorithm (FFT) is applied for the numerical inversion of the transformed quantities which in our case is the magnetic vector potential (MVP). For the inversion some criteria are used for the minimization of aliasing and truncation errors that are inherent in the numerical approach of the discrete Fourier transform (DFT) [3].

Finally eddy current density, and lift and drag forces on the moving filaments can be easily calculated as functions of space variables and of different parameters of the geometry of the problems.

2 Development of equations

Two current filaments are placed above a conducting slab of thickness \( d \). The filaments are parallel to the dividing plane \( \sigma \xi \) and move with velocity \( v = \varepsilon \xi \).

The current flowing in each filament has the analytical expression

\[
I_A = I_0 \delta(x + \alpha - vt) f_a(y - b) z_0 \tag{1}
\]

\[
I_B = -I_0 \delta(x - \alpha - vt) f_a(y - b) z_0 \tag{2}
\]

where \( \alpha > 0 \) and \( v = |v| \).

The total current of the excitation will be

\[
I = I_0 \delta(x + \alpha - vt) - \delta(x - \alpha - vt) f_a(y - b) z_0 \tag{3}
\]
Fig. 1. Geometrical configuration

where

\[ f(y - b) = \begin{cases} 1 & \text{for } y = b \\ 0 & \text{elsewhere} \end{cases} \]  

(4)

The MVP \( A = A(x, y, t) \) will fulfill the following differential equations

\[ \partial^2 A_i / \partial x^2 + \partial^2 A_i / \partial y^2 - \mu_0 \sigma \partial^2 A_i / \partial t^2 = 0 \]  

(5)

where \( i = 1, 2, 4 \) for regions 1, 2, 4 and

\[ \partial^2 A_3 / \partial x^2 + \partial^2 A_3 / \partial y^2 - \mu_0 \partial^2 A_3 / \partial t^2 = \mu_0 \partial A_3 / \partial t \]  

(6)

for region 3.

Considering the Fourier transforms of Eq. (5) and (6) with respect to \( x \) and \( t \) two ordinary differential equations are obtained with general solutions of the form,

\[ A_i = C_i e^{-\gamma t} + D_i e^{\gamma t} \]  

(7)

for \( i = 1, 2, 4 \) and

\[ A_3 = C_3 e^{-\gamma t} + D_3 e^{\gamma t} \]  

(8)

for \( i = 3 \)

\( A_i \) is the transformed function of the MVP \( A_i \) and is defined by

\[ A_i = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(x, y, t) e^{-j\omega t} e^{-j\omega x} dt \, dx \]  

(9)

The variables \( \lambda \) and \( \gamma \) are given from,

\[ \lambda = (k^2 - \mu_0 \sigma \omega^2)^{1/2}, \quad \gamma = (k^2 - \mu_0 \sigma \omega^2 + j\sigma \omega a)^{1/2} \]  

(10)

The next step is the determination of the constants \( C_i \) and \( D_i \). The transformed function \( \tilde{A}_i \) can be expressed as

\[ \tilde{A}_i = \hat{h}_i \tilde{I} \]  

(11)

where \( \hat{h}_i = \mathcal{F}_t(\mathcal{F}_x(h_i)) \) and the function \( h_i \) can be considered as response of a linear system of input \( I \) and output \( A_i \). The impulse response \( h_i \) is a function of the parameters \( y, d, b, a, \mu, \sigma \) i.e. it depends on the geometry of the problem the properties of the conducting material and the distance \( y \) at which the MVP is examined. By using Eqs. (7) and (8) for the transformed expression of the MVP and by applying the known boundary conditions in terms of the MVP [4], the constants \( C_i \) and \( D_i \) for \( i = 1, 2, 3, 4 \) are calculated (Appendix I), and the functions \( h_i \) are determined to be.

\[ h_1 = \left\{ Q((\lambda \mu - \gamma \mu_0) e^{-\gamma t} - (\lambda \mu + \gamma \mu_0) e^{\gamma t}) - \frac{\mu_0}{2\lambda} e^{-\gamma t} \right\} \times e^{-\gamma x} e^{-\gamma(y-b)} \]  

(12)

\[ h_2 = \left\{ Q((\lambda \mu - \gamma \mu_0) e^{-\gamma t} - (\lambda \mu + \gamma \mu_0) e^{\gamma t}) - \frac{\mu_0}{2\lambda} e^{-\gamma t} \right\} \times e^{-\gamma x} e^{\gamma(y-b)} \]  

(13)

where

\[ Q = \mu_0 \sigma \omega e^{-\gamma t} \left[ ((\lambda \mu - \gamma \mu_0)^2 e^{-\gamma t} - (\lambda \mu + \gamma \mu_0)^2 e^{\gamma t}) \right] \]  

(14)

The transformed function \( \tilde{I} \) of the excitation \( I \) (Eq. 3) is,

\[ \tilde{I}(k, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(x, y, t) e^{-j\omega t} e^{-j\omega x} dt \, dx \]  

(17)

Or

\[ \tilde{I}(k, \omega) = \frac{-4\pi I_0 j}{v} \sin (\omega x/v) \delta(k + \omega/v) \]  

(18)

### 3 Eddy current density calculation

Considering now that the relation between current density and MVP inside the conducting medium is \( I_s = -j\omega \sigma \tilde{A}_s \), the eddy current density is given by

\[ I_s(k, \omega, y) = \frac{-4\pi \sigma I_0 j}{v} \sin (\omega x/v) \tilde{h}_3 \delta(k + \omega/v) \]  

(19)

The two dimensional Inverse Fourier transform, which needed for the inverse transformation of \( I_s \), can be replaced by using two one dimensional Fourier transforms, expressed as

\[ I'_s(x, \omega, y) = \frac{-2\sigma I_0 j}{|\omega|} \sin (\omega x/v) \tilde{h}_3 e^{-j\omega x/v} \]  

(20)